UASA of complex models: Coping with dynamic and static inputs

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Introduction

Building energy model

Uncertainties on:
- thermophysical materials
- weather data
- user behavior, ...

⇒ Uncertainties on energy consumption

• How are the uncertainties propagated through the model?
• Which inputs are responsible for these uncertainties?

⇒ Need to analyze the uncertainties to better understand the energy consumption
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Model

Energy building model

\[ y(t, \theta) = g(\omega^d(t, \theta), \omega^s(\theta), t) \]

- \( \omega^d(t, \theta) \): uncertain weather data depending on \( t \)
- \( \omega^s(\theta) \): uncertain thermophysical properties of materials
- \( y \): energy consumption
- \( t \): time (spatio/temporal variable)
- \( g \): PDE
Energy building model

\[ y(t, \theta) = g(\omega^d(t, \theta), \omega^s(\theta), t) \]

- Thermophysical properties \( \omega^s_i(\theta) \) (static inputs)
  - Random variable
  - Marginal distribution
  - Random sampling methods

- Weather data \( \omega^d_i(t, \theta) \) (dynamic inputs)
  - Random processes
  - Mean value \( \bar{\omega}^d_i(t) \)
  - Covariance function \( C_i(t_1, t_2) \)

Problem: how to generate consistent samples of dynamic inputs?
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Generation of dynamic inputs samples

Weather data $\omega^d_i(t, \theta)$
- Random processes
- Mean value $\bar{\omega}^d_i(t)$
- Covariance function $C_i(t_1, t_2)$

Series expansion
Complete set of deterministic functions with corresponding random coefficients

Karhunen-Loève decomposition
- Eigen-decomposition of the covariance function
- Orthogonal deterministic basis functions
- Uncorrelated random coefficients
- Optimal encapsulation of information contained in the random process into a set of discrete uncorrelated random variables
Generation of dynamic inputs samples

Weather data $\omega^d_i(t, \theta)$
- Random processes
- mean value $\bar{\omega}^d_i(t)$
- covariance function $C_i(t_1, t_2)$

Karhunen-Loève decomposition

$$\omega^d_i(t, \theta) \approx \bar{\omega}^d_i(t) + \sum_{k=1}^{M_i} \sqrt{\lambda_k} \xi_k(\theta) f_k(t)$$

- $\lambda_k$ : eigenvalues of $C_i(t_1, t_2)$
- $f_k$ : eigenfunctions of $C_i(t_1, t_2)$
- $M_i$ : number of modes
- $\xi_k$ : independent normally distributed random variables
Generation of dynamic inputs samples

Weather data $\omega_i^d(t, \theta)$
- Random processes
- mean value $\bar{\omega}_i^d(t)$
- covariance function $C_i(t_1, t_2)$

Karhunen-Loève decomposition

$$
\omega_i^d(t, \theta) \approx \bar{\omega}_i^d(t) + \sum_{k=1}^{M_i} \sqrt{\lambda_k} \xi_k(\theta) f_k(t)
$$

- $\lambda_k$ and $f_k(t)$ solutions of Fredholm integral:

$$
\int_{D} C_i(t_1, t_2) f_k(t_1) dt_1 = \lambda_k f_k(t_2)
$$

- Resolution based on wavelet transform of $C_i(t_1, t_2)$
Generation of dynamic inputs samples

Karhunen-Loève decomposition

\[
\omega_i^d(t, \theta) \approx \bar{\omega}_i^d(t) + \sum_{k=1}^{M_i} \sqrt{\lambda_k} \xi_k(\theta) f_k(t)
\]

- \( M_i \) modes containing 95\% of \( V(\omega_i^d(t, \theta)) \)
- Influence of the inputs 
  \[ \{ \omega_1^d(t_1, \theta), \ldots, \omega_1^d(t_f, \theta), \omega_2^d(t_1, \theta), \ldots, \omega_2^d(t_f, \theta), \ldots \} \]
- Influence of the modes
Sensitivity analysis of the dynamic inputs

- \( N_d \) dynamic inputs \( \omega_i^d(t, \theta) \)
- \( M_i \) modes for each input \( \omega_i^d(t, \theta) \)
- In all: \( n = \sum_{i=1}^{N_d} M_i \) modes to analyze
- SA of the inputs through \( \{\xi_1, \cdots, \xi_n, \omega^s\} \)

- Sensitivity indices

\[
\begin{align*}
\{ & \{ \xi_1, \cdots, \xi_{n_1} \}, \xi_{n_1+1}, \cdots, \xi_{n_2}, \cdots, \xi_n, \omega_1^s, \cdots, \omega_N^s \} \\
M_1 \text{ modes of } & \omega_1^d & M_2 \text{ modes of } & \omega_2^d
\end{align*}
\]

Sensitivity index of \( \omega_i^d \) (grouped modes)
\[
S_1 = \frac{V(E(y|\xi_1, \cdots, \xi_{n_1})))}{V(y)},
\]

\cdots

Sensitivity index of \( \omega_i^s \)
\[
S_i = \frac{V(E(y|\omega_i^s))}{V(y)},
\]

\cdots
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Results

Initial problem

\[ y(\theta) = g(\omega^d(t, \theta), t) \]

<table>
<thead>
<tr>
<th>6 dynamic inputs</th>
<th>2 outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature ( T )</td>
<td>Heat consumption grnd floor ( y_1 )</td>
</tr>
<tr>
<td>Direct solar radiation ( D )</td>
<td>Heat consumption 1st floor ( y_2 )</td>
</tr>
<tr>
<td>Diffuse solar radiation ( d )</td>
<td></td>
</tr>
<tr>
<td>Wind speed ( V )</td>
<td></td>
</tr>
<tr>
<td>Wind direction ( V_d )</td>
<td></td>
</tr>
<tr>
<td>Relative humidity ( H )</td>
<td></td>
</tr>
</tbody>
</table>

- Daily consumption summed over one month
- Data for representative month of january
Results

Procedure

1. Generation of the dynamic inputs
   1.1 Perform a 2D wavelet transform of $C_i(t_1, t_2), i = 1, \ldots, 6$
   Fast Haar wavelet transform algorithm
   Here 3072 modes in all
   $\Rightarrow \lambda_k, f_k(t)$

1.2 Generate the independent random variables $\xi_k$

1.3 Generate the $N_d = 6$ dynamic inputs $\omega^d_i$

\[
\omega^d_i \approx \bar{\omega}^d_i(t) + \sum_{k=1}^{M_i} \sqrt{\lambda_k} \xi_k(\theta)f_k(t)
\]
Results

Procedure

2 Sensitivity analysis

2.1 Simulate the model with the dynamic inputs

\[ y(\theta) = g(\omega^d(t, \theta), t) \]

- Higher consumption at the 1st floor
- More glass surface at the ground floor
- Solar gain more important at the ground floor
- Sensitivity indices to check this assumption
Results

2.2 Sensitivity indices of the grouped modes

\[ S_i = \frac{V(E(y|x_i, \cdots, x_n))}{V(y)} \]

- Direct solar radiation more influential at the ground floor
- Temperature more influential at the 1st floor
- Importance of solar gain during winter time
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In prospect

- Study the influence of the individual modes
- Include the thermophysical properties of the materials $\omega^s$ (static inputs)
- Consider the dynamic output $y(t, \theta)$ (thermal comfort)
- Work done in the context of the french project ASenDyn granted by the CNRS
Thank you for your attention

Questions?