Abstract—Managing uncertainties in industrial systems is a daily challenge to ensure improved design, robust operation, accountable performance and responsive risk control. This paper aims to illustrate the different depth analyses that the uncertainty software LEONAR, devoted to a specific application, can propose. The physical model of LEONAR describes some of the phenomena related to the molten core behavior, which may arise in severe accidents in Pressurized Water Reactors, starting from the core degradation and ending either with stabilization or with the complete ablation of the concrete of the pit. LEONAR computes several statistical quantities (failure probabilities of the reactor vessel and pit, probability distributions of output variables) and performs sensitivity analyses. This paper presents several examples of LEONAR use: precise and punctual needs for not specialist engineers, detailed analyses for confirmed users and helps to physical model developers.

Monte Carlo simulation; uncertainty; sensitivity analysis; severe accident; nuclear power plant

I. INTRODUCTION

Nowadays, numerical modeling is more and more used to simulate complex phenomena. This is the case in the field of severe accidents that could happen on a nuclear power plant and which can induce reactor core melting, reactor vessel rupture, hydrogen combustion inside the reactor containment, corium (i.e. molten core) concrete interaction, etc. In that field, corresponding to a temperature level up to 3000 K, experiments are extremely costly, the phenomena are highly coupled with each other and the scale for the final application (plant scale) is very large. That is why numerical simulation is used to deal with reactor applications. Unfortunately, the simulation of this type of phenomena is sometimes limited by the lack of knowledge on the phenomena, on the physical parameters entering in the models and on the input data related to the scenarios of the accident. Uncertainty studies have then to be carried out in order to take into account the sources of imprecision in the use of numerical modelling [1][2].

For the purpose of such uncertainty studies on some specific scenarios of PWR (Pressurized Water Reactor) severe accident, the CEA develops the LEONAR software. LEONAR is based on the CEA uncertainty software, called URANIE, which provides a tool box that can be used for various types of physical phenomena The URANIE functionalities cover the major part of the needs for the uncertainty studies [2][3][6]: deterministic and statistical sampling design methods, uncertainty propagation, optimization and sensitivity analysis techniques, surrogate models, automatic launching of the calculations to clusters.

This paper describes and illustrates the functionalities of the LEONAR software. The plan of this paper follows the “Uncertainty” generic methodology of the ESREDA book [3]. Section II introduces briefly the physical model and the industrial background. Section III discusses the framework of our uncertainty study. Section IV presents some examples of uncertainty and sensitivity analyses done with LEONAR.

II. THE STUDY MODEL AND METHODOLOGY

A. The nuclear Pressurized Water Reactor (PWR) severe accident scenarios

Several scenarios of accident may lead to what is defined as a severe accident, characterized by the core degradation i.e. loosing its geometrical integrity by melting or debris formation. These scenarios may be a break in the primary or secondary cooling circuit, pump failure, etc. If the safety cooling systems also fail or are delayed, no more water will cool the reactor core. Consequently the temperature of the fuel rods will increase until several chemical reactions occur. At these temperature levels a very high exothermic reaction between zirconium of the rod cladding and steam occurs, which triggers the degradation phenomena. The other energy source is the decay heat due to fission products. The accumulation of fuel and cladding materials, the so called corium, first as solid debris, with no sufficient cooling device, leads to the formation of a liquid pool in the centre of the reactor core surrounded by debris. This corium pool extends progressively until it reaches the baffle. The baffle is also molten, and corium flows into the vessel bottom head. The bottom head contains some water: the corium is partly solidified, but after vaporization, and due to the decay heat, the debris again becomes liquid, and the steel of the pressure vessel wall begins to interact with the corium. If no water is present outside of the pressure vessel, the rupture of the pressure vessel occurs and the corium flows out into the reactor pit. This first part of the severe accident is called in vessel phenomena (Figure 1).
When the corium is falling into the reactor pit begins what is called *ex vessel phenomena* (Figure 1). Corium comes into contact with the concrete of the containment basemat. The corium temperature is larger than the melting temperature of the concrete. The concrete then begins to melt, and its ablation forms liquid molten oxides like silica, calcia or alumina, which incorporate the corium liquid pool, and also gas (steam and carbon dioxide) which percolate the pool. Chemical reactions occur, with oxidation of the metallic phase of the corium. Physical-chemistry effects also take place, with oxides mixtures at high temperature, crust formation at the interface between the pool and the concrete, phase segregation phenomena… The question that has to be addressed is whether and when the concrete basemat of the nuclear plant would fail since it could lead to possible release of fission product outside the containment.

![Figure 1. Schematic representation of the corium behaviour during severe accidents in PWR.](image)

The aim of the severe accident management strategy is to avoid this ultimate stage, and to stabilize the corium until its solidification after several days. One useful operation could be to inject water in the vessel or in the reactor pit during the accident. Of course the consequence of water injection depends on the injection time, the injection location (in vessel, ex vessel) and on the corium inventory.

### C. The industrial context and stake

The main question that has to be addressed in the frame of severe accident studies of nuclear power plant is whether and when radionuclides would be release outside the containment. The answer to this question depends on our ability to model the phenomena involved in severe accidents and on our strategy to mitigate the consequences of the accident (i.e. water injection). That is why many R&D efforts are dedicated to the study of core degradation, corium behavior in lower head and molten core concrete interaction. Despite the numerous progresses that have been achieved in the past years, it is to be noticed that uncertainties remain and that some of the phenomena are not correctly understood and modeled today. For a given scenario and management strategy, the time for basemat melt through is thus marred by strong uncertainties and a statistical approach enables to quantify the influence of these uncertainties. The other point is that the progression of the accident can be modified by operations such as water injection in the vessel or/and in the reactor pit. In some circumstances it can cool part of the corium, limit the mass which interacts with concrete and avoid the basemat failure. One can easily understand that the efficiency of such operation can be involved to the time at which it may occur and to the amount and mass flow rate of available water. Such quantities are not precisely known and a statistical approach can provide the range of values for which water injection might be efficient.

### B. The physical model in LEONAR

The main constraint for the physical model implemented in LEONAR is that it must run fast. For this reason, detailed models cannot be used and all the phenomena that may occur are not described. For the in-vessel part and in case of no water injection in the vessel, we use the results of the MAAP code as input data [4] to describe the core degradation for a given accident scenario. Dedicated models are implemented in LEONAR to simulate what happens in the core when water is added in the vessel, and also for the description of the corium behavior in the lower head. Classical heat transfer correlations are used to estimate the heat transfer between the corium pool in the lower head and the vessel wall. This enables the calculation of the ablation of the vessel wall and consequently the time for vessel failure and the location of the breach. Concerning the ex-vessel phenomena, we have included in LEONAR the TOLBIAC-ICB code [5] which is devoted to the simulation of the corium-concrete interaction, and which is used with a simplified physical-chemistry model. New functionalities have been added to TOLBIAC-ICB to deal with several connected rooms (reactor pit, corridor, instrumentation room, containment).
III. UNDERLYING FRAMEWORK OF THE UNCERTAINTY STUDY

A. Specification of the uncertainty study

From the user standpoint, the development of LEONAR follows several objectives:

- Carries out probabilistic assessments (by Monte-Carlo simulations) of failure of the reactor vessel and corium melt-through in the reactor pit;
- Analyzes the influence of specific actions (e.g. core reflooding or water injection in the reactor pit) and/or plant modifications (e.g. dedicated water injection system in the reactor pit);
- Integrates sensitivity tools to analyze the effects of the modeled phenomena on the computer code outputs by a large number of Monte-Carlo simulations.

Therefore, the main goal of LEONAR is to propagate scenario and model input uncertainties through a severe accident computer code. There is no formal criterion when using such a tool. Indeed, because of our lack of knowledge on the modeled phenomena, quantitative results are subject to caution. However, the obtained failure probabilities can be subsequently used, for example, to deduce which materials or actions (related to input variables) are useless for the system reliability increase. This kind of evidence would be useful when writing recommendations in the safety analysis reports. This first goal of LEONAR corresponds to the goal “Select” of the “Uncertainty” guide [3], namely to compare relative performance and optimize the choice of maintenance policy, operation or design of the system.

The ancillary goal of LEONAR is to understand the various physical phenomena occurring during a specific scenario. Indeed, the sequential nature of the accidental transient can become particularly complex. A deep analysis of one computation result is helpful for the modelers but can become costly and fastidious when treating several results. When looking at a large number of model computations (typically larger than 10), the sensitivity analysis tools are welcome. Such tools are devoted to this output understanding objective. This goal corresponds to the goal “Understand” of the “Uncertainty” guide [3]: to understand the influence or rank importance of uncertainties, thereby to guide any additional measurements, modeling or R&D efforts.

The whole framework is shown in Figure 2 and specific aspects of the framework are elaborated in the following sections. Let us first underline that, in our case, the system model is not CPU time expensive. Therefore, performing several thousands of simulation is possible in a few hours.

The system quantities of interest are the vessel failure probability, the reactor pit failure probability and the histogram of the model output variables. There are 23 output variables of interest. Two output variables are qualitative (vessel failure and reactor pit failure) and 21 output variables are quantitative:

- time of failure of vessel,
- corium mass in the core, corium mass in the bottom of the vessel and corium mass going in the reactor pit,
- times of degradation start in four different locations,
- times of degradation stop in four different locations and in the vessel,
- times of axial failure in four different locations,
- times of radial failure in four different locations.

The model has 76 inputs: 44 of them are fixed while 32 are affected by uncertainty. The uncertain inputs may be classified as:

- 12 uncertain inputs related to the water management. Two input variables follow a binary distribution: the presence/absence of water in the vessel and the presence/absence of water in the reactor pit. 10 other input variables, representing the arrival times and flow rates in five different places or premises, follow uniform distributions on large variation ranges;
- 8 uncertain inputs related to the in-vessel physical module: debris porosities, debris diameters, various fractions and factors representing coefficients applied to physical laws. All these highly uncertain input parameters are supposed to follow uniform distributions. Uncertainties placed on coefficients are a consequence of our lack of knowledge of the underlying physics;
- 12 uncertain inputs related to the ex-vessel physical module, which include mass corium and time vessel break (inactive variables in case of in-vessel

![Figure 2. LEONAR software in the common methodological framework of the “Uncertainty” guide [3].](image)
To propagate uncertainties, the standard Monte Carlo method is used by the way of Simple Random Sampling (SRS) or Latin Hypercube Sampling (LHS) [2][6]. When performing a sufficient number of simulations (typically a few hundreds), these methods allow to obtain the full probability distributions of the quantitative variables of interest. In our case, there is no problem to perform several thousands of the physical model simulations.

For the two qualitative variables of interest, i.e. the two observed failures (vessel rupture and reactor pit break), each failure probability is obtained with

\[
P_f = \frac{N_f}{N}
\]

where \(N_f\) is the number of simulations with failure and \(N\) is the total number of simulations. Using the Simple Random Sampling, the relative precision of this estimate is approximated by the classical formula:

\[
\text{cv}(P_f) = \sqrt{\frac{(1 - P_f)P_f}{N}} \frac{1}{P_f}
\]

where \(\text{cv}(P_f)\) is the coefficient of variation. When \(N\) tends to infinity \(\text{cv}(P_f)\) tends to zero. This formula has to be taken with care when the number of simulations is small.

When using sensitivity analysis methods, we have to make the distinction between qualitative and quantitative output variables of interest. For quantitative output variables, we use the well-known scatterplot graphical tools and regression coefficient values [6]. Scatterplots allow visualizing the individual effects of each input on each output while linear regression coefficients associate quantitative values. These analyses are valid under the assumption of a linear relation between the output and all the input variables. When the model is not linear, we use the rank transformation of each sample vector and the rank-regression coefficients as quantitative sensitivity indices. This method is valid under the assumption of a monotonic relation between the output and all the input variables, which is often the case in physical models. In all cases, we compute the coefficients of determination to validate/invalidate the linear/monotonic assumption [6].

Sensitivity analysis on qualitative output variables (the failure events) can be made from a random sample with standard statistical tools. First, the graphical tool of boxplot can be used to visualize the correlation between a quantitative variable \(X\) (in our case each input) and a qualitative variable \(Y\) (in our case the failure or not) [7]. We propose an illustration of this tool in section 4.2.1. Second, we can obtain in a simple way some quantitative indices using the correlation ratios [7]. If the correlation ratio is equal to zero, there is no dependence between \(X\) and \(Y\), leading to the conclusion that \(X\) is not influent for the failure. A correlation ratio close to 1 reveals a strong link between \(X\) and \(Y\), leading to the conclusion that the \(X\) values influence the failure.

D. Feedback process

The main aim of LEONAR is to compare several accidental scenarios. In order to make the studied reactor safer, this can result in modifications of the modeled system, leading to modification of the LEONAR physical model. A secondary aim is to identify the main sources of uncertainty into which further R&D efforts should be devoted. More precise determination of these uncertainties would allow gaining more confidence on LEONAR results. Non understandable results or identifying influent specific phenomena can also lead to the introduction by the modelers of new models in order to better model the physical phenomens.

IV. EXAMPLES OF USE

In this section, we illustrate with imaginary scenarios the three different levels for the use of LEONAR that we distinguish:

- The precise and punctual needs for a beginner are illustrated via failure probability computations associated with sensitivity analysis;
- The detailed analyses for confirmed users are illustrated via the results of the multiple regression analysis;
- The help to physical model developers is illustrated via the scatterplot tool.

A. Uncertainty propagation and sensitivity analysis on probabilities of failure

An important stake of LEONAR is to bring some answers about the influence of water injection inside the vessel and/or outside the vessel during the accident. We illustrate this kind of study with the following scenario:

- 22 uniform random variables with large variation ranges (see Table 1 for details),
- one binary random variable for the water presence outside the vessel (with probability 0.5 for the water presence and 0.5 for the water absence,
- the water inside the vessel is present,
- 8 other uncertain variables are kept constant at nominal values.

We compute \(N=500\) random simulations following a LHS design. Therefore, water is present outside the vessel in 250 cases and not present in 250 other cases. We distinguish the simulations with water injection and the simulations without water injection to compute in each case the failure
probability value of the reactor pit. The ratio between these two probabilities is relevant information:

\[
P(\text{failure without water injection}) = 2.5.
\]

In this example, the reduction of the probabilities due to the water injection is considered as small. Specialists consider that a real effect would give a ratio larger than 10. This kind of result can be useful for the operators to integrate such results in the safety case or to answer to the regulator questions.

<table>
<thead>
<tr>
<th>Input variable</th>
<th>Name</th>
<th>Range of the uniform law</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water arrival time in vessel</td>
<td>iaecu</td>
<td>[0;9e5] (s)</td>
</tr>
<tr>
<td>Water arrival time in pit</td>
<td>iape</td>
<td>[0;4e5] (s)</td>
</tr>
<tr>
<td>Water flow rate in vessel</td>
<td>decu</td>
<td>[0;0.015] (m²/s)</td>
</tr>
<tr>
<td>Water flow rate in pit</td>
<td>dep</td>
<td>[0;0.015] (m²/s)</td>
</tr>
<tr>
<td>Debris porosity in core</td>
<td>pdc</td>
<td>[0;1.05] (-)</td>
</tr>
<tr>
<td>Debris porosity in bottom head</td>
<td>pdf</td>
<td>[0;1.05] (-)</td>
</tr>
<tr>
<td>Fission product power proportion in vessel</td>
<td>pppc</td>
<td>[0.01;0.25] (-)</td>
</tr>
<tr>
<td>Debris diameter in core</td>
<td>ddc</td>
<td>[1e-4;4e-3] (m)</td>
</tr>
<tr>
<td>Debris diameter in bottom head</td>
<td>ddf</td>
<td>[1e-4;1e-2] (m)</td>
</tr>
<tr>
<td>Corium fraction in bottom head</td>
<td>dfs</td>
<td>[0;0.05] (-)</td>
</tr>
<tr>
<td>Limiting factor for the critical flux outside the vessel</td>
<td>ffcc</td>
<td>[0.5;1] (-)</td>
</tr>
<tr>
<td>Corium fraction in compartment</td>
<td>fmc</td>
<td>[0;1.02] (-)</td>
</tr>
<tr>
<td>Fission product power proportion ex vessel</td>
<td>ppph</td>
<td>[0.01;0.25] (-)</td>
</tr>
<tr>
<td>Heat exchange coefficient ratio</td>
<td>hsh</td>
<td>[0.5;5] (-)</td>
</tr>
<tr>
<td>Debris fraction in pit during relocation</td>
<td>fde</td>
<td>[0;0.05] (-)</td>
</tr>
<tr>
<td>Ejected corium fraction outside pit</td>
<td>fep</td>
<td>[0;0.04] (-)</td>
</tr>
<tr>
<td>Debris fraction in pit after corium ejection</td>
<td>fdp</td>
<td>[0;0.04] (-)</td>
</tr>
<tr>
<td>Ejected debris fraction outside the molten core</td>
<td>fdec</td>
<td>[0.5;1] (-)</td>
</tr>
<tr>
<td>Corium fraction passing in other location</td>
<td>fsr</td>
<td>[0;0.03] (-)</td>
</tr>
<tr>
<td>Factor for the surface spreading</td>
<td>fse</td>
<td>[0;0.1] (-)</td>
</tr>
<tr>
<td>Corium fraction for the spreading under water</td>
<td>fdet</td>
<td>[0;1] (-)</td>
</tr>
<tr>
<td>Debris porosity ex vessel</td>
<td>pdc</td>
<td>[0;1.09] (-)</td>
</tr>
</tbody>
</table>

Table 1. List of random input variables of the LEONAR scenario. For the variables variation ranges, (-) means “no unity”.

By looking at the boxplots between each input variable and the failure variable, the engineer can also identify which input variables have some effects on the failure of the reactor pit. Figure 3 shows the strong influence of the time of the water injection inside the vessel (iaecu input variable) on the probability of failure: larger values of iaecu lead to more failures (right side of the figure).

Figure 3. Boxplot of the iaecu input variable (time of the water injection inside the vessel) relative to the failure (right) / non failure (left) of the reactor pit.

B. Sensitivity analysis related to quantitative outputs: regression coefficients

For the same simulations, Figure 4 shows the sensitivity indices (based on regression coefficients) of all the input variables for the specific output value mcp (corium mass in the reactor pit). Such analysis brings relevant information about the model input parameters which strongly affect the pit mass corium: debris fraction inside the water (in case of water injection in the pit) and time of water injection inside the vessel (in case of no water injection in the pit). Consequently, the users will try to more precisely model the uncertainties on these input variables in order to reduce the uncertainty on the corium mass.

C. Analysis tools for physical model developers

Using another scenario and simulations, we illustrate at present the use of LEONAR as a help tool for the model developers. Figure 5 shows a scatterplot of several hundreds Monte Carlo simulations between the input variable flfc (limit factor of the critical flux) and the output variable mbfdc (corium mass which migrates to the vessel bottom). The strange behavior of the scatterplot (emergence of strata) has allowed detecting anomalies on the results. In fact, important phenomena related to the corium transfer were not included in the physical model.
Figure 4. For the output variable \( mcp \) (corium mass in the reactor pit), standardized regression coefficients of each input variable. Up: with water injection in the reactor pit; Bottom: without water injection in the pit.

Figure 5. Scatterplot between the input variable \( flfc \) (abscissa) and the output variable \( mbfdc \) (ordinate).

V. CONCLUSION

Uncertainty studies play an important role in assessment of complex phenomena modeling. Their primary goal is to take into account the sources of imprecision due to lack of knowledge on the phenomena, on the physical parameters entering in the models and on the input data related to the scenarios. Since a few decades, a lot of specific softwares have been developed to treat uncertainty industrial problems. For example, the URANIE software (developed by CEA) contains all the necessary tools to perform uncertainty and sensitivity analyses. URANIE can be used for various applications and in very different data processing contexts.

However, for industrial dissemination, all inclusive and integrated softwares (easy to use by non specialists) are needed, and specialized statistical tools may not answer to this requirement. For this purpose, the CEA has developed the software LEONAR, dedicated to uncertainty and sensitivity analyses of phenomena arising in severe accidents in PWR. Referring to the example of LEONAR tool, we have presented the different levels of use that an uncertainty study software can propose: precise and punctual needs for not specialist engineer, detailed analyses for confirmed user and helps to physical model developers.

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