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# PROPOSITION DE SUJET DE THESE

Intitulé : MultiDisciplinary Analysis solved by adaptive disciplinary Gaussian processes

### Référence : TIS-DTIS-2024-05

(à rappeler dans toute correspondance)

Début de la thèse : octobre 2024

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#### Mots clés

Processus gaussien, analyse multidisciplinaire

### Profil et compétences recherchées

Bac+5, ingénieur ou université

### Présentation du projet doctoral, contexte et objectif

Today's civil aviation industry is faced with the challenge of reducing its environmental impact. Among the various efforts required, the need to design and optimize a new generation of aircraft in a very short time is one of the most exciting. It is now well established that numerical simulations will play a major role in this task, and need to be massively developed for effective use right from the first design stage. To capitalize on this development, it is equally important to understand and accurately simulate the multiphysical phenomena that govern aircraft performance. Multidisciplinary Design and Optimization (MDO) is an efficient approach to take into account the interactions between the various disciplines involved in conceptual aircraft design (aerodynamic, structural mechanics, propulsion etc.). Formally these interactions involve the resolution of a non-linear system of equations (e.g. aero-elasticity) called MultiDisciplinary Analysis (MDA). During the conceptual phase, this non-linear system of equations is solved by a partitioned approach. This means that disciplinary solvers are called as black boxes by a non-linear solver until convergence is reached. Typically, a finite element (FEM) solver, handling the structural mechanics resolution, is coupled with a computational fluid dynamics (CFD) solver, handling the aerodynamic resolution, by a fixed-point algorithm.

As an illustrative example, there can be one design parameter  $X_1$  and two disciplinary solvers  $f_1(X_1, f_2)$  and  $f_2(f_1)$ , in which case the aim is to find their output values  $y_1^*(X_1)$  and  $y_2^*(X_1)$  at the equilibrium, with  $f_2(y_1^*) = y_2^*$ , and with  $f_1(X_1, y_2^*) = y_1^*$ .

This type of non-linear system is usually solved by Newton Raphson algorithm if the Jacobian matrix of the system is available or by gradient free algorithms such as the non-linear Gauss Seidel method or the non-linear Jacobi method. The convergence criterion of these methods is well established but their use in a parametric setting (such as optimization or uncertainty quantification) can be computationally expensive (using the previous illustrative example this means that the algorithm should be used each time the value of the parameter  $X_1$  changes). In order to tackle this problem, we recently proposed a heuristic approach based on the adaptive construction of disciplinary surrogate models and Bayesian paradigm (*G. Berthelin et al. (2022)*, Disciplinary Proper Orthogonal Decomposition and Interpolation for the resolution of parametrized Multidisciplinary Analysis. International Journal for Numerical Methods in Engineering)

Indeed, Gaussian process models now constitute the method of choice to tackle prediction and uncertainty related to the output(s) of physical systems. When the choice of inputs for queries is open, active (or adaptive) learning is available as a boosting feature, providing in some cases colossal improvements of the budget/accuracy ratio.

The coupled settings presented above open many theoretical and methodological perspectives for Gaussian process models. For example, the coupled systems we are interested in generally have a unique solution, given the input variables. To come back to the above illustrative example, for all  $X_1$ , there is a unique solution  $(y_1^*, y_2^*)$ . We will then aim for Gaussian process models that guarantee this property, by using regularity constraints. We will also explore iterative fixed point algorithms to approximate posterior quantities (e.g. mode, posterior mean and then posterior samples) for models.

We will design sequential strategies, where at each step, the aim is to optimally select a system to query and the corresponding query inputs. When a single system is under consideration, the problem has been widely considered, and a popular class of approaches is called Stepwise Uncertainty Reduction (SUR). We will thus provide SUR strategies in the coupled settings, with the aim of improving over other existing strategies.

### **References:**

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## Collaborations envisagées

Collaboration Institut de Mathématique de Toulouse et ONERA

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