# MascotNum2020 conference - Quantitative comparison of several methodologies used to model epistemic uncertainty associated with aleatory uncertainty in the context of structural reliability

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**Abstract:**

Any system or component that can exist is set to meet various requirements that can be from many types. It is therefore necessary to predict its performance. In the context of structural reliability, the performance of a system is assessed through a limit-state function which can be analytical or defined by a computational algorithm [1]. The limit-state function enables to assess for a given set of inputs if the system lies in the safety domain or the failure domain. In order to predict in which domain the system belongs to, it is necessary to quantify in the best possible manner the parameters that are fed to the limit-state function. These parameters reflect for example properties of the system such as geometrical or material properties but can also describe the system's environmental quantities such as loads. In most industrial cases, such quantification is confronted with the presence of uncertainty which implies that the input vector may not only be composed by deterministic values. Consequently, the uncertainty is propagated through the model which inevitably leads to an uncertain output. Uncertainty is often seperated into two distinct, yet not unrelatable, categories namely aleatory and epistemic uncertainty [2].

Aleatory uncertainty describes the intrinsic variability of a quantity and is assumed to be irreducible. The probabilistic theory has been widely developed and applied to treat this type of uncertainty. Each uncertain parameter is considered as a random variable defined by a probability distribution constructed from available data or expert knowledge. Therefore, the input being a random vector, the output of the limit-state function is also a random variable. A typical quantity of interest in structural reliability is the probability of failure which describes how likely a realization of the output random variable falls into the failure domain. The epistemic category treats the uncertainty which results from a lack of information. For example, only little data possibly resulting from biased measurements can be obtained. An other example is when contradictory judgments from different experts are noted. Therefore, the uncertainty is considered as reducible by acquiring more information. Different methodologies to model such uncertainty are present in the litterature. In this work the interval model, the convex model, the possibility theory and the probability box theory all coupled with the info-gap theory [3] are applied and compared .

The interval model is the less informative modeling as it simply assumes that the uncertain parameter is bounded by an upper and a lower bound with no notion of probability measure within the interval. The convex model enables to include dependency information between different parameters. The uncertain input vector belongs to a convex set. The multi-dimensional parallelepiped is used in this work. The possibility theory is a special case of the evidence theory (or Dempster-Shafer theory) and stands between the interval model and the probabilistic model in terms of information. Analagously to a probability distribution, a possibility distribution is

associated to the uncertain parameter. The main difference is that, instead of affecting a precise probability to each single value of the support of the uncertain parameter, a belief measure is associated to nested intervals contained in the support. The probability box theory (p-box) deals with uncertainty on a probability distribution. Two groups are distinguished namely free p-boxes and parametric p-boxes. In both cases, the true distribution function is supposed to belong to an envelope defined by an upper and a lower distribution function and that is the only hypothesis made with the free p-boxes case. The parametric case assumes that the type of the probability distribution is known but that its parameters are uncertain and belong to intervals. The info-gap decison theory adds the notions of robustness and opportuneness by building around a nominal value nested convex sets parametrized by an unbounded horizon of uncertainty. Instead of evaluating a quantity of interest for a specific degree of uncertainty, it shows how the worst (robustness) and best (opportuneness) possible performances vary as a function of the horizon of uncertainty.

The different types of modeling mentioned above are, for most part, closely related to each other and can be equivalent in specific cases. The variety of proposed methods that is not limited to the ones considered in this work can make it confusing for an analyst. The proposed work aims at bringing clarification by comparing theoretically and numerically the different approaches. The comparison starts by listing the input information needed to apply each modeling and ends by analysing the value of the information obtained on a specific quantity of interest. Systems with analytical limit-states are considered with hybrid uncertainty meaning that some input parameters are caracterized by a fully known probability distribution while other parameters face epistemic uncertainty and are modeled by one of the already mentioned modelings. Building such hybrid models is made possible by applying random sets theory which is a compatible framework to treat the different types of modelings together [4]. The quantity of interest is a probability of failure and more precisely a range of probabilities of failure. The notion of horizon of uncertainty found in the info-gap theory is used to compare the quantity of interest for different levels of uncertainty on the input parameters. Therefore, in addition to offering a comparison framework for different types of modeling, this work also shows that many uncertainty modelings can use the info-gap theory in order to make robustness analyses. An example of the application of info-gap in structural reliability is found in [5] in which the robustness to uncertainty of penstocks is evaluated by using nested parametrized probability boxes.

## References

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**Short biography** – After two years of preparing the entrance examination for engineering schools, I joined the mechanical engineering school IFMA (now named SIGMA Clermont). I ended my studies with an internship at EDF R&D which led to my PhD thesis in partnership with the institute FEMTO-ST in Besançon. The objective of the thesis is mainly to investigate the use of the info-gap method for structural reliability.