Back and Forth Nudging for data assimilation in geophysics

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Data Assimilation is the ensemble of techniques combining in an optimal way (in a sense to be defined) the mathematical information provided by the equations of the model and the physical information given by the observations in order to retrieve the state of a flow [1]. There are two large classes of methods: variational algorithms (4D-VAR) and sequential techniques (Kalman filtering).

The standard nudging algorithm is a simple data assimilation technique: it consists in adding to the state equations of a dynamical system a feedback term, which is proportional to the difference between the observation and its equivalent quantity computed by the resolution of the state equations. The model appears then as a weak constraint, and the nudging term forces the state variables to fit as well as possible to the observations. This is known in control theory as being the Luenberger's observer. First used in meteorology, the nudging method has been applied with success in oceanography.

The back and forth nudging algorithm, introduced in [2], consists in solving first the forward nudging equation and then the direct system backwards in time with a feedback term which is opposite to the one introduced in the forward equation. This term stabilizes this backward resolution, which is usually ill-posed for these irreversible geophysical systems. The "initial" condition of this backward resolution is the final state obtained by the standard nudging method. After resolution of this backward equation, one obtains an estimate of the initial state of the system. These forward and backward resolutions (with the feedback terms) are repeated until convergence of the algorithm.

This algorithm has been tested for various systems in geophysics [3], such as Lorenz system, viscous Burgers equation, quasi-geostrophic model , or shallow water equations [4] and compared with 4D-VAR method. The convergence of this algorithm has been studied for linear transport equations and non-linear Burgers equation, with or without viscosity in [5]. An improvement to the Back and Forth Nudging (BFN) algorithm for handling diffusion in the context of geophysical data assimilation, in which the sign of the diffusion term is changed in the backward integrations, has been introduced in [6] and the convergence of this algorithm has been studied, in particular for linear transport equations. This modified BFN has been applied to Burgers equations and compared with other algorithms [7].

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