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Title:

Lipschitz-Killing curvatures of excursion sets for two-dimensional random fields

(joint work with H. Biermé, C. Duval and A. Estrade)

Abstract:

Lipschitz-Killing (LK) curvatures are geometrical tools which permit to analyse d dimensional objects. Considering a black and white image in dimension $d = 2$, there are three LK curvatures: the surface area, the half perimeter and the Euler characteristic. Each of them brings a distinct information on the geometry of the black (*resp.* white) zone. The surface area is related to its occupation density, the perimeter to its regularity and the Euler characteristic to its connectivity.

Considering real-life data as a realization of a random field and analyzing them with the help of Lipschitz-Killing curvatures is an effective approach that has already been successfully applied in various disciplines in the past decades. For instance, in cosmology, the question of Gaussianity and anisotropy of the Cosmic Microwave Background radiation has been tackled in a huge amount of publications. A similar methodology is also applied for analyzing the distribution of galaxies. Another domain where the LK curvatures are used as methodological tools is in medical imaging as, for example, brain imaging or mammograms. In this context, the main purpose could be to find the locations of high brain activity or to study the breast density.

In our papers, we study the three Lipschitz-Killing curvatures for the excursion sets of a 2-dimensional standard (centered and unit variance) stationary isotropic random field X. These geometrical characteristics can be estimated without bias if the field satisfies a kinematic formula, such as a smooth Gaussian field or some shot noise fields.

If the field is Gaussian, we show how to remove the constraining assumption that the field is standard. To cope with the unknown location and scale parameters of X, we introduce novel fundamental quantities called *effective level* and *effective spectral moment* and we propose unbiased and asymptotically normal estimators of these parameters. Finally, we use these tools to built a test to determine if two images of excursion sets can be compared. This test is applied on both synthesized and real mammograms. Meanwhile, we establish the consistency of the empirical variance estimators of the third LK curvature under a weak condition on the correlation function of X.

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