Bayes factors for multinomial goodness-of-fit testing

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Abstract:

Classical significance tests with fixed dimension are known to over-reject point null hypotheses in large samples due to the high levels of power they attain as the acceptance region shrinks with sample size for a given significance level. Relevant discussions on this issue include [1], [3], [5], and [7]. The need to find alternatives is evident in the present-day context, as the utility and the correct interpretation of classical tests is being debated, and there is concern about the frequent misuse and misinterpretation of classical measures of evidence (see, for example, [2], [9], [12], and [13]). Bayes factors, used extensively in both Bayesian model selection and hypothesis testing, are a popular alternative to classical significance tests. Bayesian methods, however, require the specification of prior distributions, and Bayes factors are known to depend rather strongly on prior distribution specification. This dependency does not disappear as the sample size increases, as it does in parameter estimation. Moreover, non-informative priors are often defined only up to a constant multiple, yielding indeterminate Bayes factors. To solve this indeterminacy, alternatives based on training samples have been suggested, including: fractional Bayes factors [10], intrinsic Bayes factors [4], expected-posterior priors [11], power-expected-posterior priors [8], and integral priors [6]. Using the aforementioned methods, we will derive default Bayes factors and prior distributions to be used in multinomial point null hypothesis testing, and assess and compare their properties and their performance on simulated data, as we intend to extend the application of default model selection methods to the problem of multinomial point null hypothesis testing.

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Short biography – PhD candidate in Applied Mathematics for Economics and Management with two publications in peer-reviewed journals and experience in the banking industry. Conducting research in the fields of Bayesian model selection and hypothesis testing with funding from the Portuguese Foundation for Science and Technology. Holder of a postgraduate degree in Statistical Systems, a master's degree in Applied Econometrics, and an undergraduate degree in Economics.