Sequential incrementation of the dimension in computer experiments

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Abstract:
Industrial products are studied numerically before being sold. The corresponding numerical codes involve geometrical or environmental inputs and physical outputs. They are complex as they involve lots of input variables. Studies are provided to quantify the influence of the inputs on the behavior of the outputs. They usually use metamodels based on a set of already simulated cases (called a Design of Experiments DOE). The first studies focus on a small amount of important inputs, letting them vary freely to see their influence on the output, while the other inputs are fixed to nominal values. Then, the studies are complexified as some previously fixed inputs are progressively involved.

The classical way of treating the increasing number of inputs is to start from scratch each time new inputs are released, regenerating a Design of Experiment and a metamodel independent from the previous ones. This approach can be time consuming and the previous data is lost because unused.

An alternative, presented in our work, is to update gradually the design and the metamodel, based on the previous ones. This enables to exploit all the available data. Our case study is the fan from the car engine cooling system. In this simplified low dimensional case provided by Valeo, there are three inputs: the flowrate, the blade pitch, and the blade chord. The output to predict is the Pressure Loss. We consider three cases:

- Firstly, only one of the inputs can vary and the others are fixed. For this case, we dispose of a DOE and a corresponding kriging metamodel.
- Then, one of the two previously fixed inputs is freed. We must create a new design of experiment and a new metamodel depending on the two free inputs but using the previous information (metamodel and DOE).
- Same thing when the third input is released.

We choose to model the output by a Gaussian process $Y$. The three inputs are denoted by $x_1$, $x_2$, and $x_3$ and belong to $[0, 1]$. We respectively denote by $a$ and $b$ the values at which $x_2$ and $x_3$ are fixed before they are freed. We decompose $Y$ as follows:

$$ Y(x_1, x_2, x_3) = \begin{cases} 
Y_0(x_1) & \text{if } (x_2, x_3) = (a, b) \\
Y_1(x_1, x_2) & \text{if } x_3 = b \\
Y_2(x_1, x_2, x_3) & \text{else}
\end{cases} $$

$$ Y_0(x_1) = Z_0(x_1) $$
$$ Y_1(x_1, x_2) = Y_0(x_1) + Z_1(x_1, x_2) $$
$$ Y_2(x_1, x_2, x_3) = Y_1(x_1, x_2) + Z_2(x_1, x_2, x_3) $$
Figure 1: Different designs in 2D with good projections on the previous 1D design: unoptimized (on panel 1a), optimal for maximin (on panel 1b), optimal for distance to previous design (on panel 1c), and optimal for a combination of the two (on panel 1d)

with $Z_0$, $Z_1$, $Z_2$ independent Gaussian processes. The definition of $Y$ implies:

$$
\begin{align*}
Z_1(x_1, a) &= 0 \quad \forall x_1 \in [0, 1] \\
Z_2(x_1, x_2, b) &= 0 \quad \forall (x_1, x_2) \in [0,1]^2
\end{align*}
$$

We propose two types of nonstationary processes that satisfy the previous condition, both based on stationary Gaussian processes $\tilde{Z}_i$ (associated with $Z_i$). For $Z_1$, the proposed models are the following:

- $Z_1(x_1, x_2) = \Psi_1(x_2 - a)\tilde{Z}_1(x_1, x_2)$ with $\Psi_1$ a function null at 0.
- $Z_1(x_1, x_2) = [\tilde{Z}_1(x_1, x_2) | \tilde{Z}_1(x_1', a) = 0, \forall x_1' \in [0,1]]$, following the work of Bertrand Gautier [?].

We then propose sampling strategies that enable the decoupling of $Z_0$, $Z_1$, and $Z_2$ in the likelihood. For example samples to train $Y_1$ that enable the decoupling of $Z_0$ and $Z_1$ are shown in figure 1. Their construction is based on the simulated annealing routine for maximin LHS from DiceDesign [?].

We compare our methodology to a classic kriging metamodel on analytical test functions and on the Valeo test case.

**Short biography** – My name is Thierry Gonon. I am at the end of my first year of phd. I previously was an engineer student at Ecole Centrale de Lyon. My last internship at the school took place at Valeo in the Thermal Systems branch. I was in charge of implementing kriging tools (Prediction, Confidence intervals, EGO...) in their software to treat their numerical CFD
simulations. At the end of my internship, we decided to continue with a phd in collaboration between École Centrale de Lyon and Valeo.