Transient dynamics of three-dimensional beam trusses under impulse loads

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2 Abstract

Spatial structures are often subjected to impulse loads which induce high-frequency (HF) wave propagation, as in the pyrotechnic solar panel unfolding for satellites or the so-called pyrotechnic cut (separation of the launcher and the payload). These loads can damage the integrity of the equipments attached on the structure. Despite some recent researches, the characterization of the transient response to such loads remains an open problem. Thus it is necessary to improve the prediction of HF structural responses with efficient theoretical and numerical models. The objective of this research is to develop a reliable model of the HF energy evolution within three-dimensional beam trusses in order to predict, for example, their potential steady-state behavior at late times or the energy paths.
In the HF range, classical modal analyses are not applicable on account of the small wavelength and high modal density. Moreover the effect of the heterogenities can not be neglected in this range because their characteritic size are closed to the wavelength. Therefore two strategies have been developed by engineers: either global approaches like the Statistical Energy Analysis (SEA) [1] which gives no local information about the energy paths, or local approaches like the Vibrational Conductivity Analogy (VCA) [2]. The main issue concerning the existing local approaches is their extension to complex structures. However the theory of micro-local analysis of wave sytems shows that the energy density associated with their HF solutions satisfies radiative transfer equations [3]. In this model, heterogeneities are introduced along the same lines as done by Savin in [4]. Here they are modeled by fluctuations of the material properties of the medium represented by second order stochastic processes. Then the energy density $E(\boldsymbol{x},t)$ of the high frequency elastic waves reads:

$$E(\boldsymbol{x},t) = \sum_{\alpha} \int_{\mathbb{R}^3} w_\alpha(\boldsymbol{x},\boldsymbol{k},t)d\boldsymbol{k},$$

where the so-called specific intensity $w_\alpha(\boldsymbol{x},\boldsymbol{k},t)$ in phase space $(\boldsymbol{x},\boldsymbol{k}) \in D \times \mathbb{R}^3$ is the energy density estimator for the mode α in a specific direction $\hat{\boldsymbol{k}} = \frac{\boldsymbol{k}}{|\boldsymbol{k}|}$. They satisfy the radiative transfer equations [3]:

$$\partial_t w_\alpha + \{\lambda_\alpha, w_\alpha\} + \Sigma_\alpha w_\alpha = \sum_{\beta=1}^{M} \int_{\mathbb{S}^{d-1}} \sigma_{\alpha\beta}(\boldsymbol{x}, \mid \boldsymbol{k} \mid, \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{p}}) w_\beta(\boldsymbol{x},\boldsymbol{p},t) d\Omega(\hat{\boldsymbol{p}}).$$

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Here $\lambda_\alpha(\boldsymbol{x},\boldsymbol{k})$ is the eigenfrequency of a mode α, $\{f,g\} = \nabla_{\boldsymbol{k}} f \cdot \nabla_{\boldsymbol{x}} g - \nabla_{\boldsymbol{x}} f \cdot \nabla_{\boldsymbol{k}} g$ is the usual Poisson bracket, $\boldsymbol{x} \in D$ is the position in the bounded domain $D \subset \mathbb{R}^3$ occupied by the structure, $\boldsymbol{k} \in \mathbb{R}^3$ is the wave vector, $\sigma_{\alpha\beta} \geq 0$ is the scattering cross-section that gives the rate of conversion of a mode β coming from a direction $\hat{\boldsymbol{p}}$ to a mode α in a direction $\hat{\boldsymbol{k}}$ at a position $\boldsymbol{x}$ and a wavenumber $\mid \boldsymbol{k} \mid$, and Σ_α is the total scattering cross-section for the polarization α.
The case of HF waves in Timoshenko beams has been investigated at first. The analysis exhibits two energy modes in the HF range: a longitudinal mode and a transverse mode, each of multiplicity three for the combined compressional/bending motions and torsional/shear motions respectively. But the study of the HF Lamb spectrum of a cylindrical waveguide shows that the Timoshenko theory is not relevant for high wavenumber and so, a new model is derived from the asymptotic analysis of this spectrum [5]. At the junctions between substructures, the energy flow is partly reflected and partly transmitted. The corresponding reflection/transmission coefficients are derived along the same line as done in [6] for assemblies of two-dimensional beams or plates. The derivation of power flow transmission/reflection coefficients consistent with a three-dimensional formulation is also presented.
For the numerical resolution of transport equations, continuous finite element method can not be used on account of the discontinuities of the energy density field at the junctions. Thus a spectral discontinuous Galerkin finite element method (DG-FEM) [7] is implemented in which the numerical fluxes are derived from of the physical fluxes at each side of the interfaces. The spatial discretization is achieved by Legendre polynomials and time integration is performed with a strong stability-preserving Runge-Kutta method [8]. DG-FEM has the advantage to be weakly dissipative and weakly dispersive depending on the order of interpolation. Thus it is well suited for long times simulation in order to exhibit a possible steady-state behavior. Numerical simulations using the DG-FEM are presented for the example of a three-dimensional beam truss. The analysis of these results shows that the steady-state behavior of the truss at late times corresponds to a diffusive behavior as assumed in the Statistical Energetic Analysis [1] of structural-acoustic systems. An application of the proposed method is time reversal that can be useful for nondestructive evaluation.

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