# Reliable Based Design Optimization using k-sigma method and local sensitivity


**Benoit Delinchant - Grenoble Electrical Engineering Lab**

G2ELab - ENSE3, 38402 Grenoble, France, Benoit.Delinchant@g2elab.grenoble-inp.fr



**Abstract— This paper presents a method and a tool for solving the reliability-based design optimization (RBDO). The RBDO aims to minimize a cost function by changing the value of design parameters while ensuring a level of reliability. Uncertainties propagation is a key concept in reliable studies and it can be associated with a sensitivity analysis in order to sort parameter influences. Global and local sensitivities are compared in this study in order to keep a reasonable cost versus accuracy ratio. A software tool has been also developed to automate reliable studies. It is applied to the reliable optimization of a magnetic nano switch with SQP algorithm using Jacobian calculated by composition of automatic and symbolic differentiation.**


## Reliable Based Design Optimization

### The RBDO problem is formulated as equation (1) where :

- F : the objective function *(system performances, manufacturing cost…)*
- X : design parameters subject to uncertainties *(system dimensions, physical properties…)*
- H : inequality constraints to be satisfied *(performances, cost constraints…)*

**Optimization method:** *deterministic gradient based optimization algorithm*

**Variance based sensitivity analysis :** *from stochastic (1) to deterministic (2) domain*

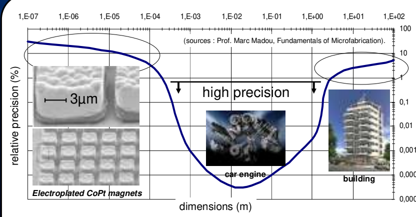
**Normal law:** *$k_\sigma$=2, 3, 4 correspond to 97.7%, 99.87%, 99.997 % of reliability*

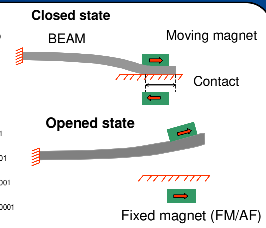
**Solution proposed:** $\sigma_{H_j(x)}$ *is proposed to be linearly approximated using symbolic differentiation and automatic differentiation.*

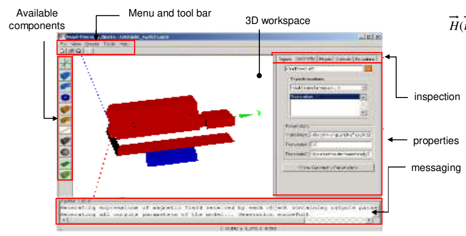
$$(1)\quad \begin{cases} \text{- minimize} & F(X) \\ \quad\text{with} & X=\{x_1,x_2,\ldots,x_n\}\in\Re^n \\ & x_i^{min}\le x_i\le x_i^{max}\quad i=1,\ldots,n \\ \text{- subject to} & P(H_j(X)\le 0)\ge P_{Hj},\ j=1,\ldots,m \end{cases}$$

$$(2)\quad \begin{cases} \text{- minimize} & F(X) \\ \quad\text{with} & x=\{x_1,x_2,\ldots,x_n\}\in\Re^n \\ & x_i^{min}\le x_i\le x_i^{max}\quad i=1,\ldots,n \\ \text{- subject to} & H_j(x)+k_\sigma.\sigma_{H_j(x)}\le 0,\ j=1,\ldots,m \end{cases}$$

## MEMS Modeling and Design




$$\vec{H}(r)=\iint_S \frac{\sigma_s.(\vec{r}-\vec{r'})}{|\vec{r}-\vec{r'}|^3}ds+\iiint_V \frac{\sigma_v.(\vec{r}-\vec{r'})}{|\vec{r}-\vec{r'}|^3}dv$$

$$\vec{F}=\iint_S \sigma_s.\vec{H}_{ext}ds+\iiint_V \sigma_v.\vec{H}_{ext}dv$$

$$\vec{B}(r)=\frac{\mu_0}{4\pi}\iiint_V \frac{\vec{j}\times(\vec{r}-\vec{r'})}{|\vec{r}-\vec{r'}|^3}dv$$

$$\vec{F}=\mu_0.\iiint_V \vec{j}\times\vec{H}_{ext}dv$$

$$\vec{F}=\frac{\mu_0}{2}.\iint [\chi]\vec{n}.\left|\vec{H}_{ext}\right|^2 ds$$


B. Delinchant et al., "Gradient based optimization of semi-numerical models with symbolic sensitivity: Application to a ferromagnetic MEMS switch device", International Journal of Applied Electromagnetics and Mechanics, Vol.30, N.3-4, pp.189-200, 2009

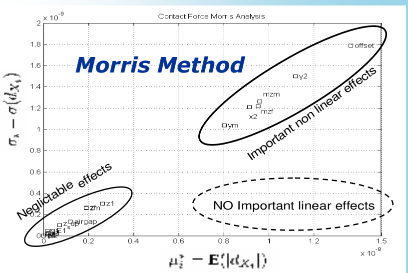

**Table 1. Optimization specifications**

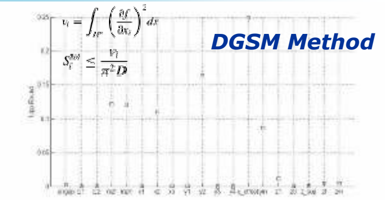
| | | | |
|---|---|---|---|
| **Objective** | V_magnet | Magnet volumes | To minimize |
| **Constraints** | Lcontact | Contact length | ≥300 nm |
| | Fcontact | Contact force | ≥1E-8 N |
| **Design variables** | Ym | Fixed and moving magnet length | [100 : 1000] nm |
| | Zf | Fixed magnet high | [100 : 1000] nm |
| | Zm | Moving magnet high | [100 : 1000] nm |
| | Y_offset | Magnet position on the beam | [2500 : 3500] nm |
| | Mzm | Mobile magnet magnetization | 1 |
| | Myf | Fixed magnet magnetization | 1 |
| | Airgap | Air gap | 50 |
| | Zsup | Substrat between fixed magnet and contact surface | 10 |
| | E1 | Magnet Young modulus (Ru-FeMn-FeCo) | 4.47x1011 |
| | E2 | Beam Young modulus (Pt) | 2.1x1011 |
| | B1,B2,… | Beams sizes B1(x1, y1, z1), B2(x2, y2, z2), … | |

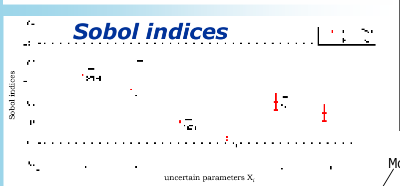
## Sensitivity Analysis

### Screening – Global sensitivity

- Screening : DGSM or Morris method
- GSA : Sobol indicies



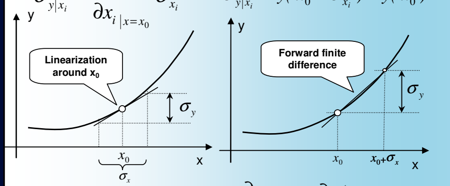



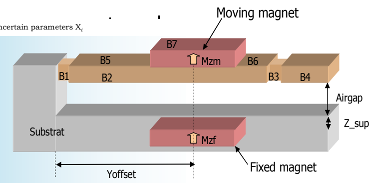




### Variance Jacobian computation

- standard deviation is obtained using linear approximation
- but gradients are required for optimization

$$\sigma_{y|x_i}=\left.\frac{\partial y}{\partial x_i}\right|_{x=x_0}\sigma_{x_i}\qquad \sigma_{y|x_i}=y(x_0+\sigma_{x_i})-y(x_0)\qquad \sigma_y=\sqrt{\sum_{i=1}^{n}\sigma_{y|x_i}^2}$$




$$\frac{\partial\sigma_y}{\partial x_j}=\frac{1}{2\sigma_y}\sum_{i=1}^{n}\left(2\sigma_{y|x_i}\frac{\partial\sigma_{y|x_i}}{\partial x_j}\right)=\frac{1}{\sigma_y}\sum_{i=1}^{n}\left(\sigma_{y|x_i}\frac{\partial\sigma_{y|x_i}}{\partial x_j}\right)$$

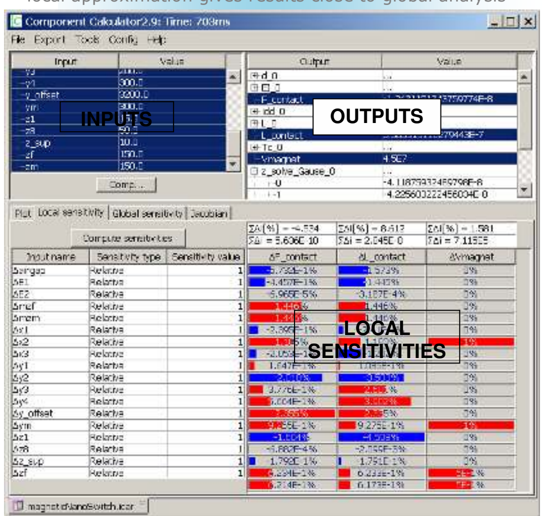
$$\frac{\partial\sigma_{y|x_i}}{\partial x_j}=\frac{\partial y(x_1,\ldots,x_i+\sigma_{x_i},\ldots,x_n)}{\partial x_j}-\frac{\partial y(x_1,\ldots,x_i,\ldots,x_n)}{\partial x_j}=J_k[j].(1+\frac{\partial(\sigma_{x_i})}{\partial x_j})-J_0[j]$$

- $J_0$ is the matrix at $X_0=(x_1,\ldots,x_n)$
- $J_k$ is the Jacobian matrix at $X=(x_1,\ldots,x_i+\sigma_{xi},\ldots,x_n)$

$$\frac{\partial(\sigma_{x_i})}{\partial x_j}=\begin{cases}\sigma_{x_i\%} & \text{if } j=i, \text{with } \sigma_{x_i\%} \text{ is relative standard deviation } [\%] \\ 0 & \text{else}\end{cases}$$

### Local sensitivity

- local standard deviation is approached by local sensitivity
- local approximation gives results close to global analysis



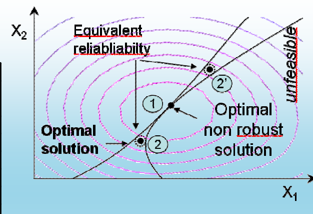

## RBDO Results

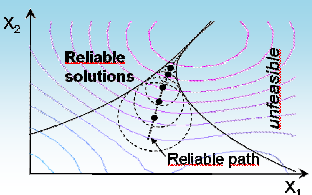
**Table 2. Optimization results**

| | Initial value | Non reliable optimale solution | Reliable optimization solutions | | |
|---|---|---|---|---|---|
| | | | $k_\sigma$=2 | $k_\sigma$=3 | $k_\sigma$=4 |
| Reliability | | 50% | 97,7% | 99,87% | 99,997% |
| Iterations | | 13 | 13 | 16 | 14 |
| Optimization time | | 24,4[s] | 219[s] | 280[s] | 262[s] |
| Ym [nm] | 500 | 330,01 | 529,80 | 690,73 | 927,34 |
| Zf [nm] | 500 | 208,95 | 276,01 | 314,11 | 359,40 |
| Zm [nm] | 500 | 208,24 | 274,64 | 312,24 | 356,75 |
| Y_offset [nm] | 3000 | 3500 | 3500 | 3500 | 3500 |
| Contact length [nm] (≥300) | | **300** | **[300**, 398.52] | **[300**, 438.3] | **[300**, 474.4] |
| Contact force [10-8N] (≥2) | | 2.115 | [2.87, 4.618] | [3.27, 6.672] | [3.71, 9.71] |
| V_magnet [1E-20 m3] (to minimize) | 25,0 | 6,884 | 14,59 | 21,63 | 33,21 |

**Table 3. Optimal solution validated with Monte Carlo simulation**

| | MC µ | MC σ | $k_\sigma$=2 (based on MC) | $k_\sigma$=2 (from optim) |
|---|---|---|---|---|
| Contact length [nm] (≥300) | 347.6 | 25.06 | [**297.48**, 397.72] | [300, 398.52] |
| Contact force [10-8N] (≥2) | 3.6 | 0.42 | [2.76, 4.44] | [2.87, 4.618] |




## Conclusions

- **We have proposed an implementation of RBDO based on $k_\sigma$ constraints and a local sensitivity approximation. It allows to compute standard deviation as well as its Jacobian to perform gradient based RBDO.**
- **Our methodology and tool has been successfully applied on the reliable design of a magnetic MEMS switch.**