Sensitivity analysis for the study of a tire model with correlated parameters and an arbitrary distribution

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1. The overview

Introduction

Particular interest has been brought last years to the study of tire-ground interface to make vehicles more safe and comfortable. Depending on the application area, many models have been developed in the literature, to describe the behavior of the tire on the ground. These models contain numerous dependent parameters with an arbitrary distribution.

Aim

- To determine the parameters affecting the variation of the tire forces.

3. Methodology

Input decorrelation using Cholesky decomposition

$$U = L^{-1}X$$

step(1)

Orthonormal data basis construction in terms of non-central statistical moments of Variables

$$P_j^{(k)}(u_j) = \sum_{i=0}^{k} p_{i,j}^{(k)} u_j^i$$

$$\begin{bmatrix} \mu_{0,j} & \mu_{1,j} & \cdots & \mu_{k,j} \\ \mu_{1,j} & \mu_{2,j} & \cdots & \mu_{k+1,j} \\ \vdots & \vdots & \cdots & \vdots \\ \mu_{k-1,j} & \mu_{k,j} & \cdots & \mu_{2k-1,j} \\ 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} p_{0,j}^k \\ p_{1,j}^k \\ \vdots \\ p_{k-1,j}^k \\ p_{k,j}^k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$\mu_{i,j} = E[u_j^i]$$

step(2)

Calculation of sensitivity indexes from polynomial chaos coefficients

$$y \approx \sum_{j=0}^{\infty} c_j \psi_j (u_1, \cdots, u_n)$$

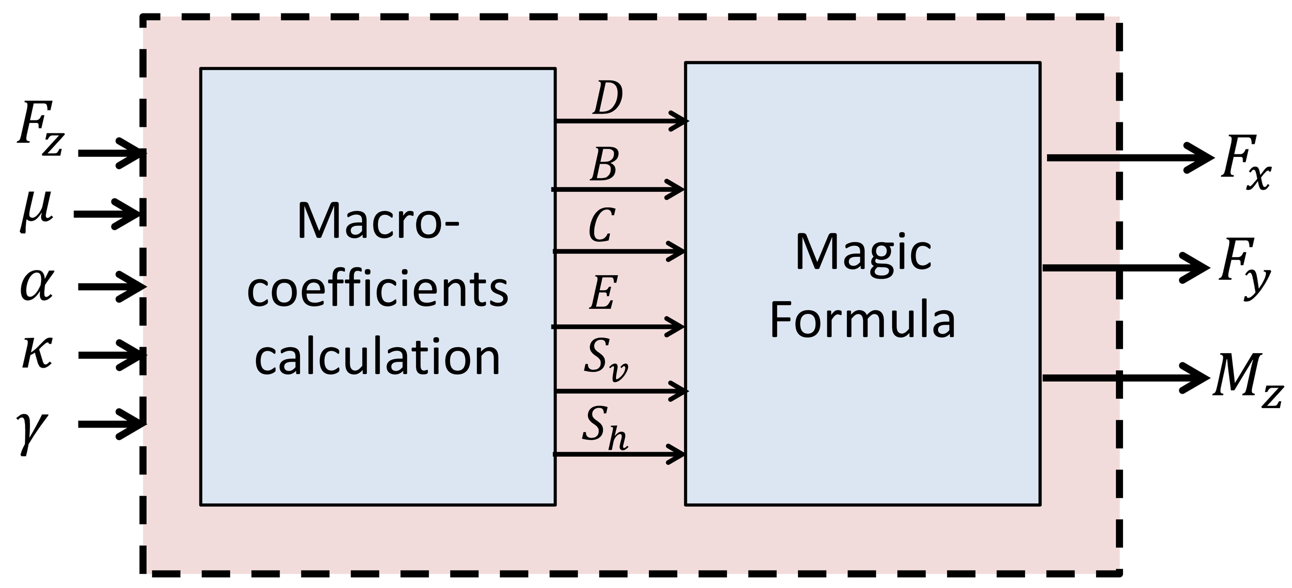
$$\psi_j(u_1, \dots, u_n) = \prod_{k=1}^{n} P_k^{\alpha_k^j} (u_k)$$

$$\widehat{S_i} = \frac{\sum_{j \in \Gamma_i} \hat{c}_j^2 E\left(\psi_j^2(u_i)\right)}{\sum_{j=1}^{M} \hat{c}_j^2 E\left(\psi_j^2(u_1, \cdots, u_n)\right)}$$

step(3)

2. Model

Model



- $[F_z, \mu, \alpha, K, \gamma]$ input variables
- $[D, B, C, E, S_v, S_h]$ macro-coefficients
- $[F_y]$ the pure lateral force considered as the output model here

$$F_y = \mu F_z \sin\left(C \arctan\left(E \left(\begin{matrix} B(\alpha + S_h) - \\ B(\alpha + S_h) - \\ \arctan(B(\alpha + S_h)) \end{matrix} \right) \right) \right) + S_v$$

- The lateral stiffness K as a function of the vertical load F_z and the camber angle γ is defined as

$$K = p_1 F_z sin\left(2arctan\left(\frac{F_z}{p_2 F_z} \right) \right) (1 - p_3|\gamma|)$$

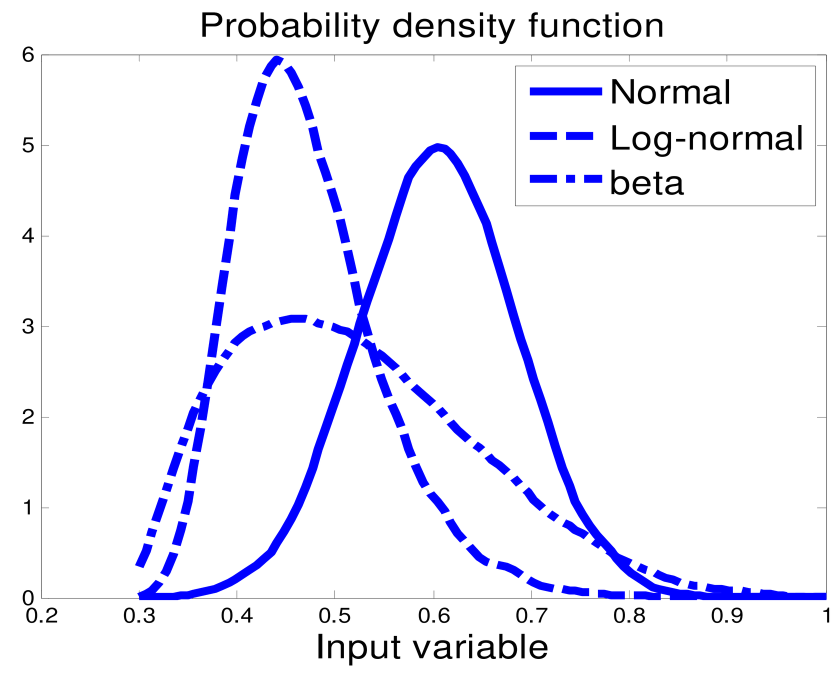
Assumption

- Dependent parameters

	α	F_z	γ	μ	K
α	1	0	0	0	0
F_z	0	1	0	0	-0,98
γ	0	0	1	0	0,07
μ	0	0	0	1	0
K	0	-0,98	0,07	0	1

Input variable correlation matrix

- Arbitrary distributed parameter



Arbitrary probability density functions of Input variable μ

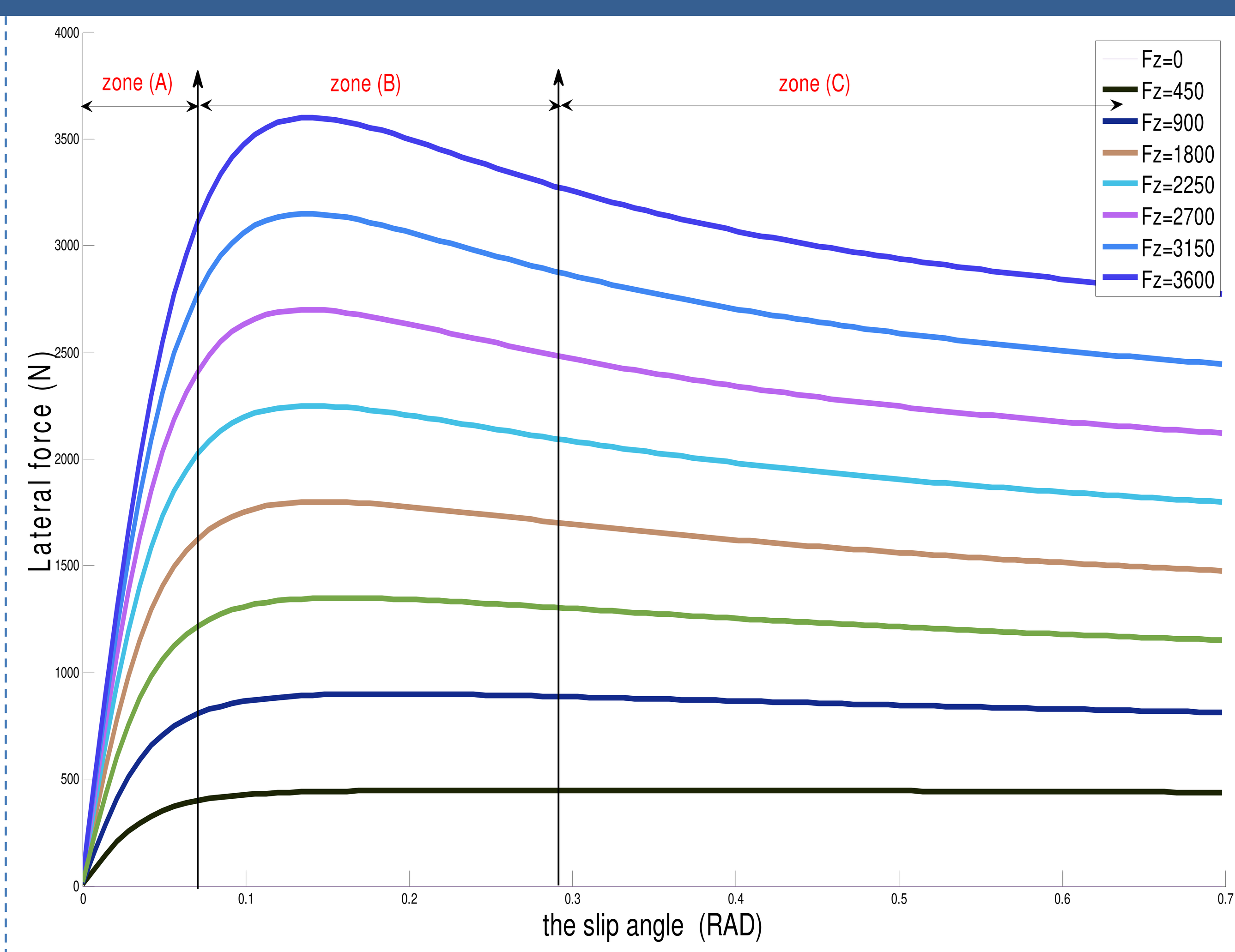
4. Results

Linear zone (A)

S_α 0.04	$S_{F_z-\alpha}$ 0.67	$S_{\gamma-F_Z\alpha}$ 0.00	$S_{\mu-\gamma F_Z\alpha}$ 0.01	S_{u_K} 0.25
S_{F_z} 0.62	$S_{\gamma-F_z}$ 0.00	$S_{\mu-\gamma F_z}$ 0.07	$S_{K-\mu\gamma F_z}$ 0.00	S_{u_α} 0.25
S_γ 0.00	$S_{\mu-\gamma}$ 0.07	$S_{K-\mu\gamma}$ 0.61	$S_{\alpha-K\mu\gamma}$ 0.25	$S_{u_{F_Z}}$ 0.02
S_μ 0.07	$S_{K-\mu}$ 0.62	$S_{\alpha-K\mu}$ 0.24	$S_{F_z-\alpha K\mu}$ 0.00	S_{u_γ} 0.00
S_K 0.61	$S_{\alpha-K}$ 0.24	$S_{F_z-\alpha K}$ 0.004	$S_{\gamma-F_Z\alpha K}$ 0.00	S_{u_μ} 0.07

Non linear zone (B,C)

S_α 0.00	$S_{F_z-\alpha}$ 0.71	$S_{\gamma-F_Z\alpha}$ 0.00	$S_{\mu-\gamma F_Z\alpha}$ 0.25	S_{u_K} 0.00
S_{F_z} 0.71	$S_{\gamma-F_z}$ 0.00	$S_{\mu-\gamma F_z}$ 0.25	$S_{K-\mu\gamma F_z}$ 0.00	S_{u_α} 0.00
S_γ 0.00	$S_{\mu-\gamma}$ 0.26	$S_{K-\mu\gamma}$ 0.69	$S_{\alpha-K\mu\gamma}$ 0.00	$S_{u_{F_Z}}$ 0.01
S_μ 0.25	$S_{K-\mu}$ 0.70	$S_{\alpha-K\mu}$ 0.00	$S_{F_z-\alpha K\mu}$ 0.01	S_{u_γ} 0.00
S_K 0.70	$S_{\alpha-K}$ 0.00	$S_{F_z-\alpha K}$ 0.01	$S_{\gamma-F_Z\alpha K}$ 0.00	S_{u_μ} 0.25



Tire lateral force F_y as a function of slip angle α , for different values of F_z

- S_{x_i}: the correlated and uncorrelated contribution of x_i
- $S_{x_i - x_j}$: the contribution of x_i without its correlative contribution with x_j
- Su_{x_i}: the proper contribution of x_i

5. Conclusion and perspectives

Conclusion

- Sensitivity analysis method proposed for model with dependent and arbitrary distributed parameters
- Application on tire model : physically consistent results were obtained

Perspectives

- Dynamic sensitivity analysis of tire model

References

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