Multidimensional Global Sensitivity Analysis for Aircraft Infrared Signature Models with Dependent Inputs

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SAMO 2013
Outline

• Context
• Metamodel: PLS Regression
• DOE choice
• Sensitivity indices
• Results
• Concluding remarks
Aircraft IRS

- Context: optimization of a **multispectral** optronics sensor
  - Sensor must detect aircraft far ahead
- Computer program CRIRA => aircraft IRS according to **aircraft properties**
  - Weather conditions
  - Attack profiles

The Infrared & Electro-Optical Systems Handbook, Vol 7, Countermeasure Systems

Several contributions to IRS: heat source emission

- **Airframe reflected light from background**
Uncertainty on input data for a given scenario

- Several data types:

  - Fixed data or parameters defined by scenario
  - Uncertain data:
    - IR optical properties of surfaces
    - Flight conditions - aspect angles
    - Atmospheric conditions

Take IRS dispersion into account to estimate optronics sensor properties

**Sensitivity Analysis:** most important input variables to acquire as a priority

Dispersion on vectorial outputs: IRS integrated in 5 up to 20 elementary bands
Constraints for Sensitivity Analysis

26 input variables: 23 continuous – 3 categorical

**Four constraints:**
1. Number of simulation runs must be small (<1000)
2. Some **qualitative inputs:** MODEL, CLOUDS, IHAZE
3. Some **correlated quantitative inputs:** RH, TA, HBASE
4. **Correlated multidimensional outputs** (5 up to 20)

⇒ Use of a metamodel based on a small computer experimental design ⇒ estimation of sensitivity indices

Many input variables + correlations + multidimensional outputs
⇒ Choice of PLS regression for this study
• **Context**

• **Metamodel: PLS Regression**

• **DOE choice**

• **Sensitivity indices**

• **Results**

• **Concluding remarks**
Metamodel

Constraints 2-3-4 are taken into account by the use of a PLS regression metamodel

Here we assume that the outputs $Y$ can be approximated by an incomplete polynomial model of degree 3 $M$, with well chosen monomials:

26 variables – **381 monomials** specified by the infrared signature expert
- the 23 original continuous inputs,
- their 23 squared terms,
- their 23 cubic terms,
- all the 253 two-input interaction terms between continuous inputs
- the 3 categorical inputs coded with their (0/1)-indicator variables
- 54 two-input interactions terms between categorical and continuous inputs

$\Rightarrow$ for a simulation run $i$ with input variables $X_i = (X_{i,1}, ..., X_{i,26})$: $Y_i = M(X_i) + \gamma_i$

$\gamma_i$ is a metamodel error
PLS Regression

PLS regression = bilinear method for relating \( d \) inputs to \( N \) outputs (Tenenhaus, Gauchi, Menardo 1995)

\[ Y_{I \times N} = X_{I \times d} \beta_{d \times N} + \epsilon_{I \times N} \]

Principle: carry out a PCA of the set of inputs \( X_j, j=1..d \) subject to the constraint that the orthogonal principal components \( t_i \) are as explanatory as possible of the set of outputs variables \( Y_k, k=1..N \)

\[ \Rightarrow \text{NIPALS iterative algorithm (Tenenhaus 1998)} \]

The significant number \( H \) of principal components is obtained thanks to a specific cross-validation test (Lazraq, Clément, Gauchi 2003)

\[ \Rightarrow \text{We obtain} \quad \hat{Y}_i = X_{i \times d} \hat{\beta}_{d \times N} \]

And thus

\[ Y_i = \hat{M}(X_i) + \epsilon_i + \gamma_i \]
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DOE Choice

Constraint 1 is taken into account thanks to the construction of a small D-optimal computer experimental design $P$

Step 1: We consider a large ~ 20000 network of candidate simulations (space filling desing):

- LHS for all quantitative variables except for meteorological ones
- non parametric reconstruction for the distribution of VIS and CLOUDS
- discrete distribution for the qualitative variables MODEL and IHAZE

- RH, TA and HBASE correlated => distribution estimation from 3200 measured data
  Non parametric kernel reconstruction of marginal distributions + dependence modeling based on a Normal copula $C$ (Nelsen 2006) using OpenTURNS

Cumulative distribution fonction $C\left(F_1\left(X_{RH}\right), F_2\left(X_{TA}\right), F_3\left(X_{HBASE}\right)\right)$

$F_1, F_2, F_3$ marginal distributions estimated by Kernel Smoothing

$$C\left(u_1, u_2, u_3\right) = \Phi_\Sigma^3\left(\Phi^{-1}\left(u_1\right), \Phi^{-1}\left(u_2\right), \Phi^{-1}\left(u_3\right)\right)$$

$u_i$ uniform law on [0,1]

$\Phi$ normal CDF – dim 1

$\Phi_\Sigma^3$ multivariate normal CDF
Step 2: We build a sequence of D-optimal designs with size $N_p$ increasing from 381 (monomials nb) up to 1000

**D-optimal design**: $\max \det(X^TX)/N_p^{381}$ - obtained thanks to Fedorov exchange algorithm

The size of the final design $P$ is chosen as a trade off between a moderate size, and a good value of the normalized determinant of the information matrix $X^TX$ associated to $M$

$\Rightarrow$ **Final size = 400**

We perform the 400 simulations, collect the multivariate outputs $Q_2 \sim 0.7$ for the different outputs – ok for Sensitivity Analysis to improve for estimation of IRS dispersion

$\Rightarrow$ **Estimation of sensitivity indices**
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SIVIP sensitivity indices

VIP (Variable Importance Projection) statistics (Tenenhaus 1998):

\[ Rd(Y, t_h) = \frac{1}{N} \sum_{k=1}^{N} cor^2(Y_k, t_h) \]

\[ Rd(Y, t_1, ..., t_H) = \sum_{h=1}^{H} \frac{1}{N} \sum_{k=1}^{N} cor^2(Y_k, t_h) \]

For \( H \) crossvalidated components a VIP is defined for each monomial \( X_j \)
It represents the monomial contribution to \( Y \) variance

\[ VIP_{Hj} = \left[ \frac{381}{Rd(Y, t_1, ..., t_H)} \sum_{h=1}^{H} Rd(Y, t_h) w_{hj}^2 \right]^{1/2} \]

Interesting property: \( \sum_{j=1}^{381} VIP_{Hj}^2 = 381 \)
SIVIP sensitivity indices

1. ISIVIP (Individual Sensitivity Indices VIP) for each monomial $X_j$

$$ISIVIP_j = \frac{VIP_{Hj}^2}{381}$$

$$\sum_{j=1}^{381} ISIVIP_j = 1$$

2. TSIVIP Total Sensitivity Indices for each original input variable:

$$TSIVIP_i = \sum_{u=1}^{J} ISIVIP_{\Omega_{iu}}$$

$u^{th}$ index set where the $i$ index is present (total number of such set $= J$)
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Results: integrated and multispectral IRS

Integrated IRS: About **10 really important variables** – 7 first associated to meteorological conditions
The 4 most important are meteorological ones
Multispectral IRS: Same 7 most important variables
some differences / integrated IRS for the 2 over important ones (SALB – E_ver)

=> If we want to reduce IRS uncertainty, we can combine the optics sensor with some detectors that can measure these atmospheric data
Results: 2 elementary bands

TSIVIP (%)

Same 7 most important variables in all configurations, but the rankings are different for the 2 elementary bands

😊 Very easy to consider different selections and merging of bands
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Concluding remarks

- Simple and effective approach well-adapted to our 4 constraints, when the computer simulation can be approximated by a polynomial of degree $\leq 3$ (no large nonlinearity)
  
  R package to appear (person to contact: JP Gauchi)

- Enable to select **10 variables among 26** that have an important impact on IRS variability

- Sensitivity Analysis simultaneously in 10 spectral bands
  
  Very easy to consider different selections and merging of bands

⇒ Carry through the specification of a multispectral sensor:

  **metamodel with 10 input variables - uncertainty propagation** $\Rightarrow$ IRS dispersion

Work in progress:

- Adaptive construction of the design of experiments
- Adaptive selection of the monomials
Questions ?

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