

# ***Contribution of Statistics to Human Exposure Assessment***

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Mascot Num 2014

ETH Zurich 2014 April 25<sup>th</sup>



# outline



Wireless communication today

Determinist approach of Exposure and EMF

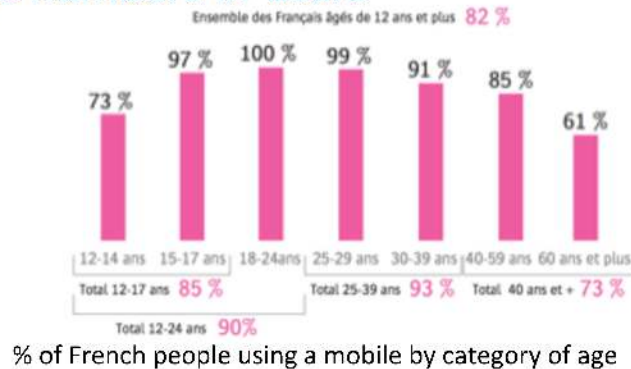
The challenge of the variability for EMF

Surrogate models in EMF exposure assessement

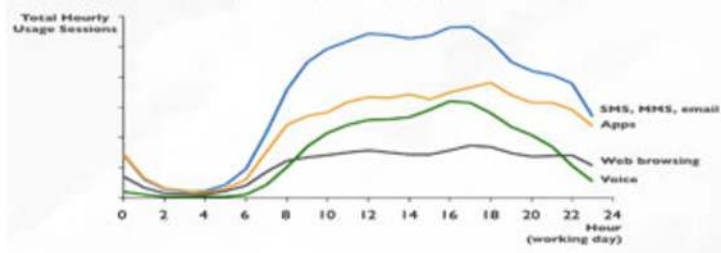
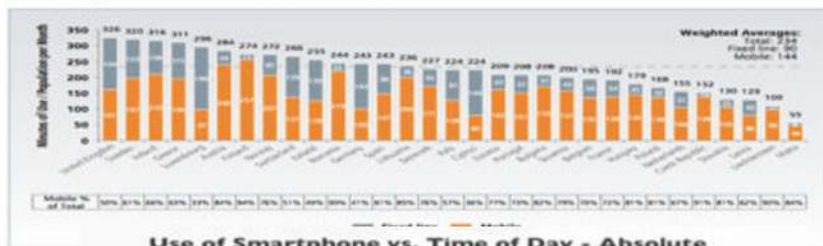
# Intensive use of wireless communication systems



- Large numbers of users



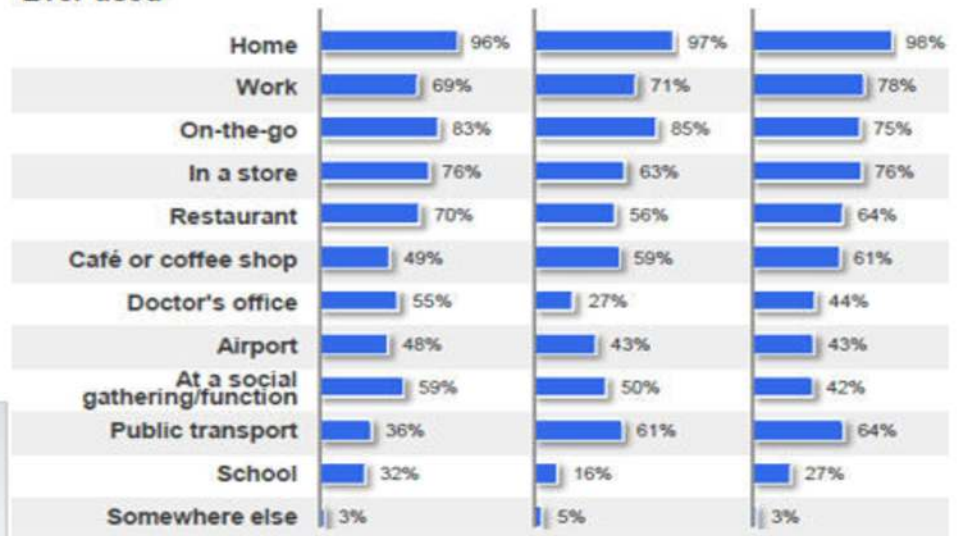
- Larger and larger duration of use



- versatile use

- IPSOS pooling in 2012

Ever used

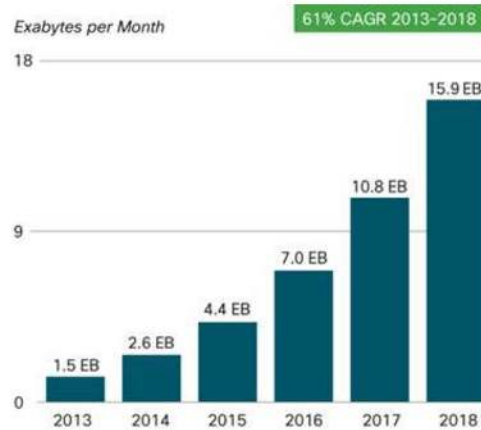


internet mobile:  
 2011: 189 Mo per month.  
 2012 342 Mo per month



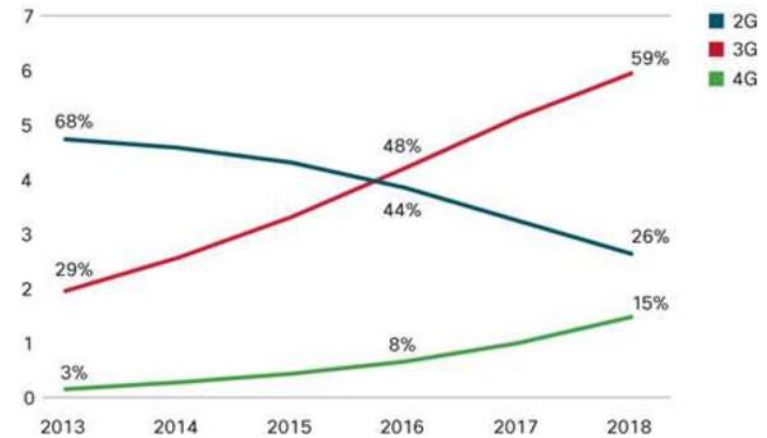
# Wireless Communication: Trends

- Data exchange increase
- Networks evolve



Source: Cisco VNI Mobile, 2014

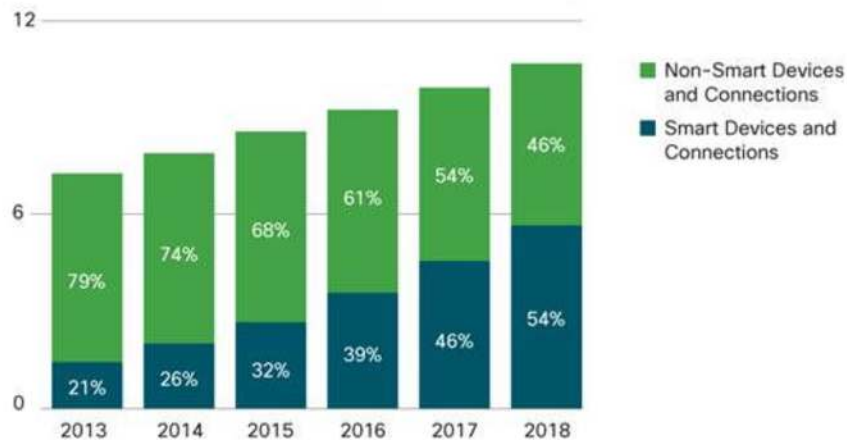
Billions of Devices or Connections



Source: Cisco VNI Mobile, 2014

Billions of Devices

8% CAGR 2013-2018



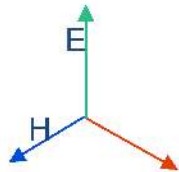
Percentages refer to device or connections share.

Source: Cisco VNI Mobile, 2014



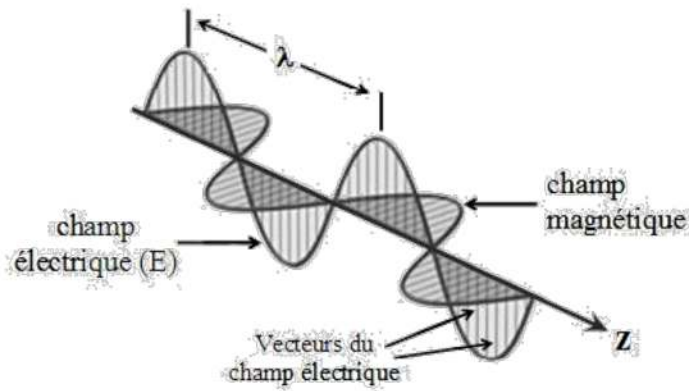


# Electromagnetic Fields (EMF) and Wireless

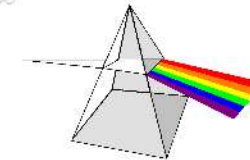
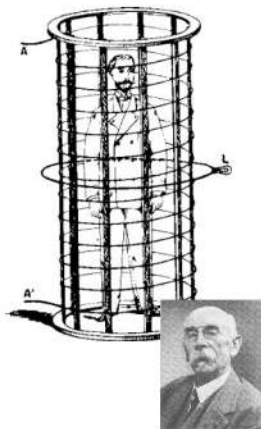


EMF Wave is composed of Electric (E) and magnetic fields (H)

In free space  
 $E/H=377\Omega$



The information is conveyed through the energy of the electromagnetic wave





# Risk perception even if there is no evidence of sanitary effect

- Large research effort conducted since 15 years
  - World Health Organisation EMF project
  - Worldwide research effort
  - Several EU and National projects and programs
  - Protection limits exist
  - No evidence of sanitary effect below the legal limits (ICNIRP limits)

ICNIRP limits

Basic restrictions	Public	Workers
Whole body SAR (W/kg)	0.08	0.4
Local SAR (W/kg) Head - Trunk	2	10
Local SAR (W/kg) Limbs	4	20

LA GAZETTE DE BRUXELLES  
 Actualités Articles Dossiers Liens Réserve  
 Accueil du site > Articles > Mobilisation contre une antenne relais à Etterbeek  
 MONDE SANS FIL  
 Mobilisation contre une antenne relais à Etterbeek  
 mardi 10 septembre 2013, par La rédaction



## But risk perception

- Euro-barmoter 2010 :
  - 70% say that mobile phone masts have some effects on health.
  - 67 % think that mobile telephones have some effects on their health.



# Compliance to limits and Risk Perception

- Compliance is fundamental but not enough today
- Assess, limit and monitor the exposure is key



Common lab of Orange Labs and Mines Telecom Institute.



## Whist Lab

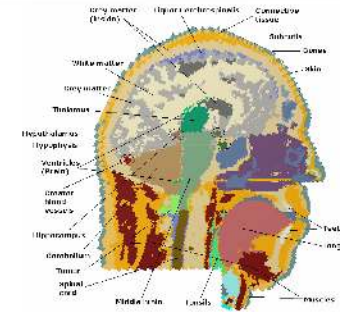
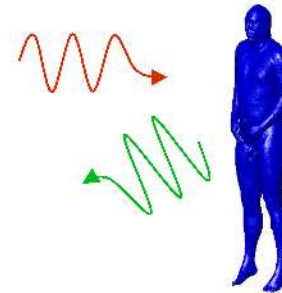
Laboratoire commun de l'Institut Télécom et de Orange Labs



# EMF Human Exposure: Absorption

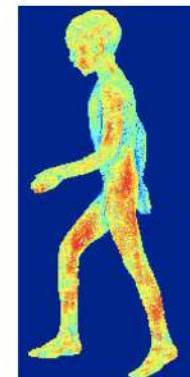
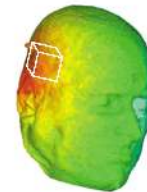
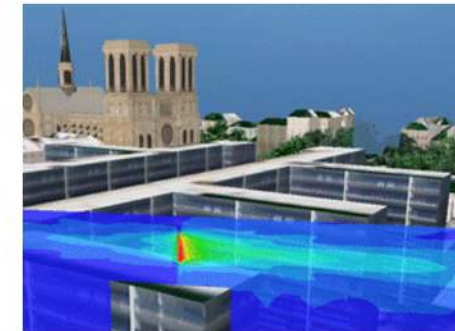


- The absorption depends on
  - Shape
  - Tissues
  - Frequency



## Exposure matrix

- E, H
- .... But also SAR
- The SAR characterizes the power deposited in tissues. It quantifies the absorbed power per mass unit
- The SAR is often averaged over the whole body or over a small mass (eg 1 or 10 g)



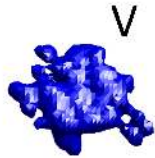
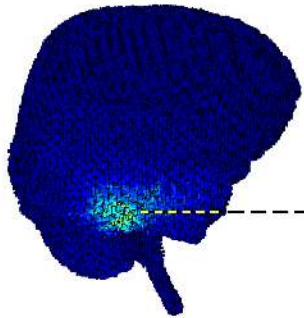
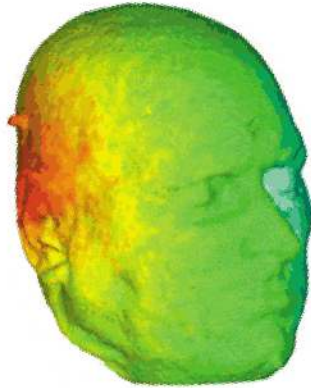
$$SAR = \frac{d\left(\frac{dW}{dm}\right)}{dt}$$





# Specific Absorption Rate ( SAR )

The SAR is linked to the electric field and to the conductivity



$$P_{abs\ in\ V} = \frac{1}{2} \iiint_V \sigma \mathbf{E}^2 dv$$

$$SAR = \frac{\sigma E^2}{2\rho}$$

$$SAR_V = \frac{P_{abs\ in\ V}}{M_V}$$



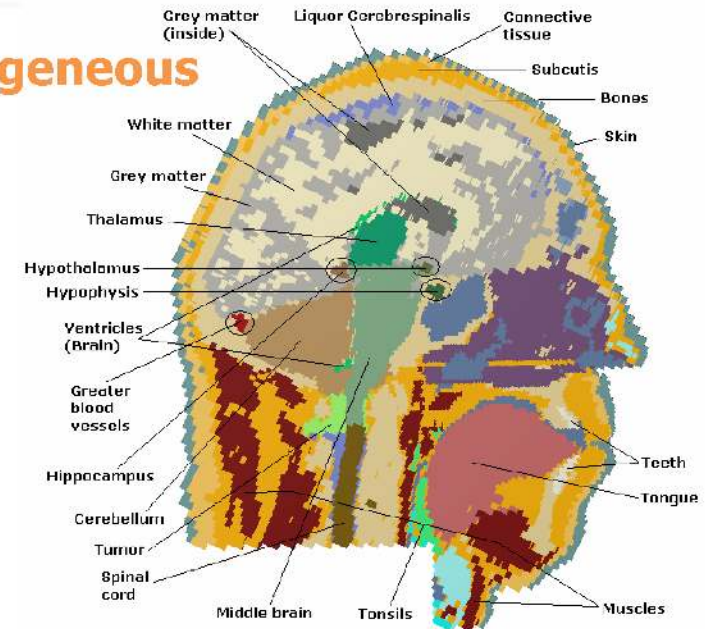
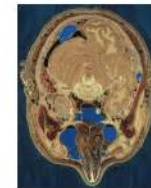
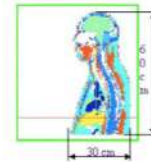
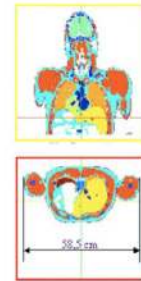
# Human body : heterogeneous, dispersive and lossy tissues

The field propagation and energy absorption is strongly influenced by heterogeneous tissues

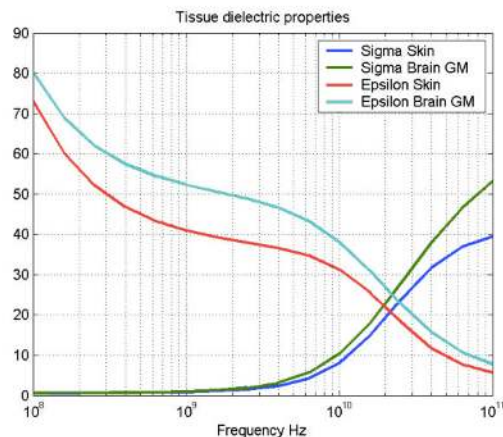


Dispersive and Lossy

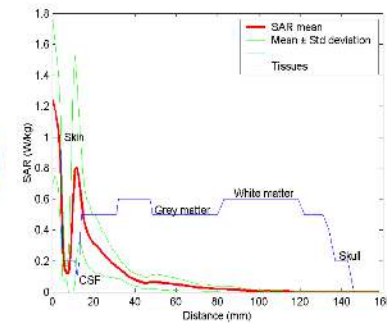
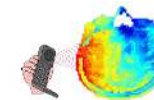
heterogeneous



F= 900 MHz



Tissue	Epsilon	Sigma
Blood	61.3	1.53
Bone_Cortical	12.4	0.14
Bone_Marrow_Infiltrated	11.2	0.22
Bone_Marrow_Not_Infiltrated	5.5	0.04
Cartilage	42.6	0.78
Cerebro_Spinal_Fluid	68.6	2.41
Eye_Tissue(Sclera)	55.2	1.16
Fat	5.4	0.05
Grey_Matter	52.7	0.94
Muscle	55.0	0.94
Nerve(Spinal_chord)	32.5	0.57
Skin(Dry)	41.4	0.86
Skin(Wet)	46.0	0.84
Tongue	55.2	0.93
White_Matter	38.8	0.59





# « Deterministic » dosimetry is mature

Large effort since 20 years

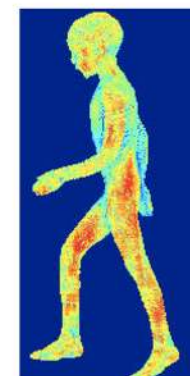
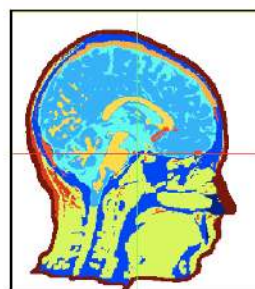
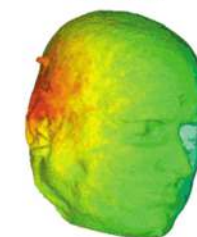
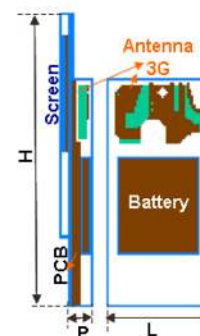
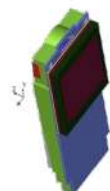
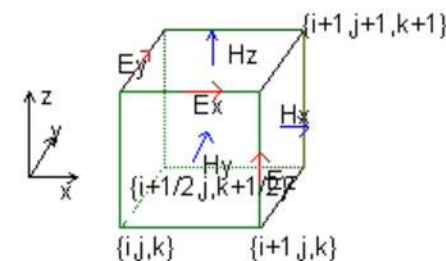
- **Experimentally.** Probe and protocols have been developed and implemented in standards
- **Numerically:** with HPC, GPU, simulations are larger and larger, faster and faster



CENELEC



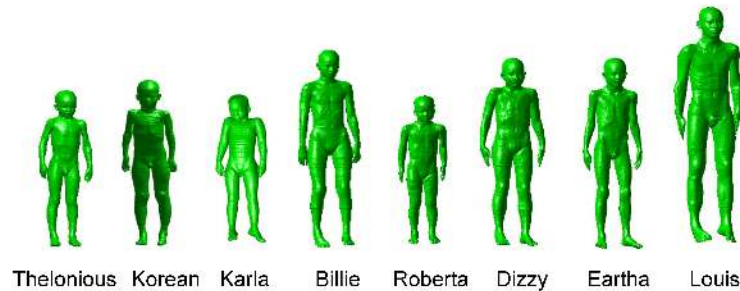
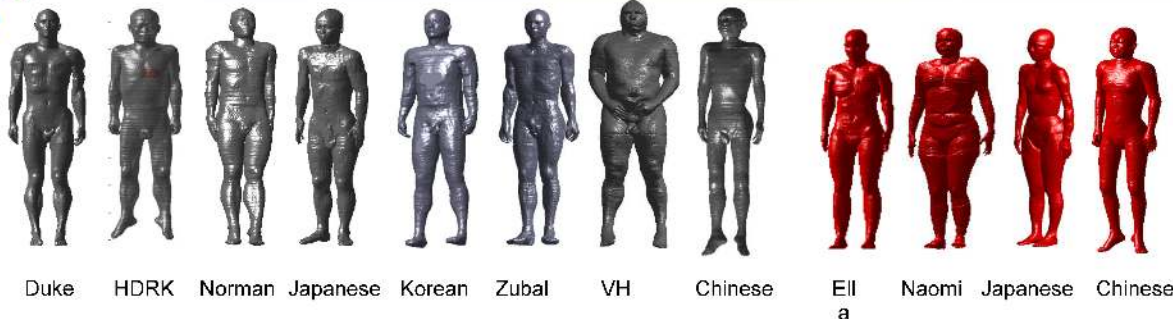
FDTD







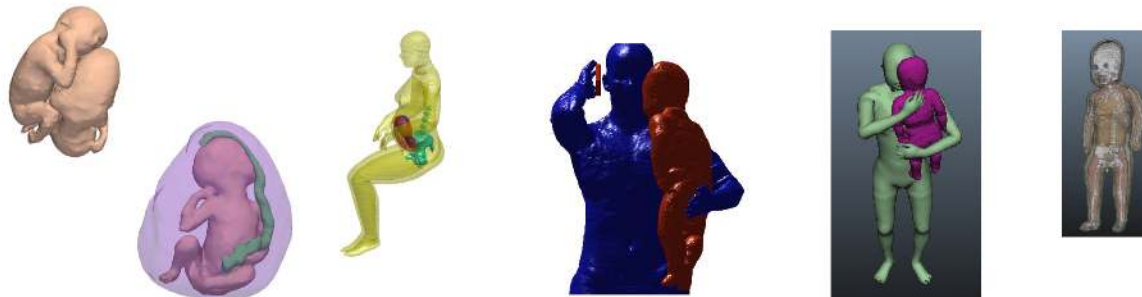
# Voxel body models exist



Even if the total number is limited and even if the models have not been randomly selected

The representativeness of the existing phantom is not guarantee

Develop a new phantom requests months (acquisition, segmentation, validation)





# Numerical SAR assessment

**In Bio-electromagnetism, the FDTD (Finite Difference in Time Domain) is the most popular method to solve the Maxwell PDE**

$$\text{rot} \vec{E} = - \frac{\partial(\mu_0 \vec{u}_r * \vec{H})}{\partial t}$$

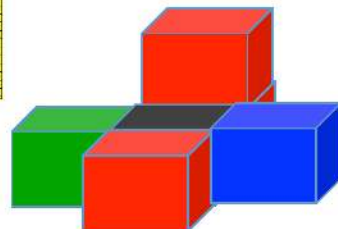
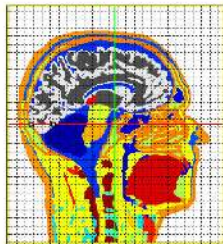
Solve the Maxwell PDE over an orthogonal grid

$$\text{rot} \vec{H} = \frac{\partial(\epsilon_0 \vec{\epsilon}_r * \vec{E})}{\partial t} + \vec{J} + \vec{D}$$

Explicit formulation does not require any matrix inversion

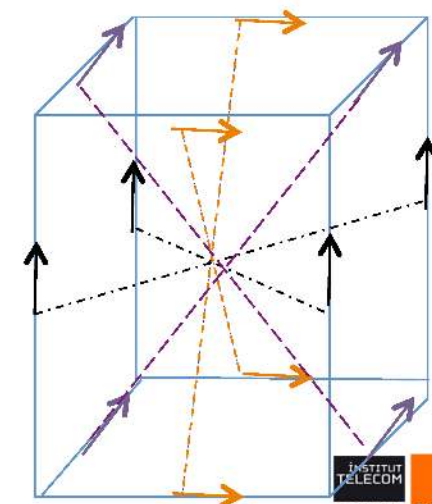
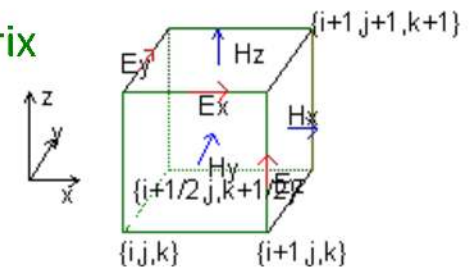
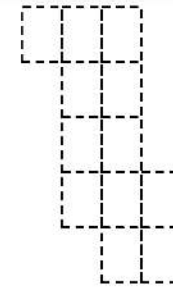
## Finite Difference

$$\frac{\partial E}{\partial x} = \mu \frac{\partial H}{\partial t} \Rightarrow \frac{E^{n\Delta t} - E^{(n+1)\Delta t}}{\Delta x} = \mu \frac{H^{(n-1/2)\Delta t} - H^{(n+1/2)\Delta t}}{\Delta t}$$



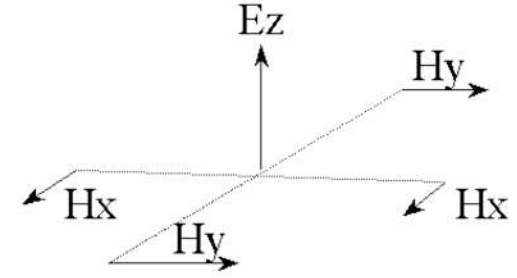
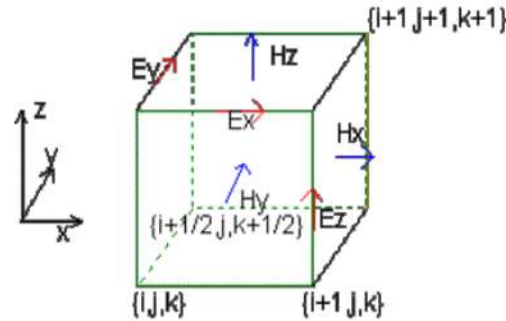
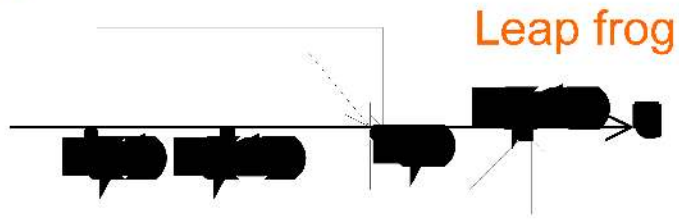
$$P_{abs} = \frac{1}{2} \iiint \sigma E^2 dv$$

$$SAR = \frac{\sigma E^2}{2\rho}$$





# FDTD: Leap frog and Stability

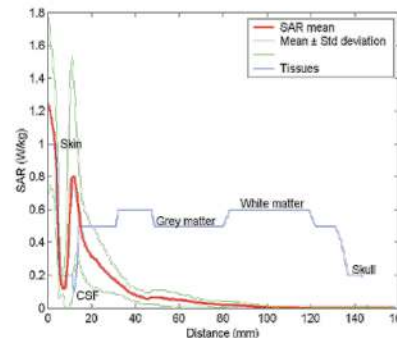
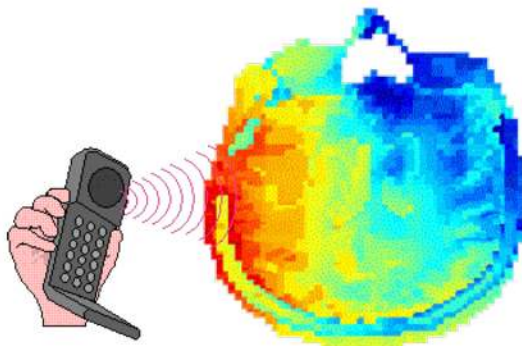


$$E_x^n(i-1/2, j, k) = E_x^{n-1}(i-1/2, j, k) + \frac{\Delta t}{\epsilon_0 \epsilon_r} \cdot \frac{H_z^{n-1/2}(i-1/2, j+1/2, k) - H_z^{n-1/2}(i-1/2, j-1/2, k)}{\Delta y} - \frac{\Delta t}{\epsilon_0 \epsilon_r} \cdot \frac{H_y^{n-1/2}(i-1/2, j, k+1/2) - H_y^{n-1/2}(i-1/2, j, k-1/2)}{\Delta z}$$

stability

$$dt \leq \frac{1}{v \sqrt{\frac{1}{dx^2} + \frac{1}{dy^2} + \frac{1}{dz^2}}}$$

Divide the resolution by 2  
the time computation by 2







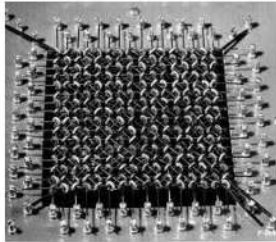
# Why FDTD

- Pro
  - Easy to use
  - No matrix inversion
  - Voxel models can be easily used
- Cons
  - Stair case
  - Time computation





# Easy but not without constraints



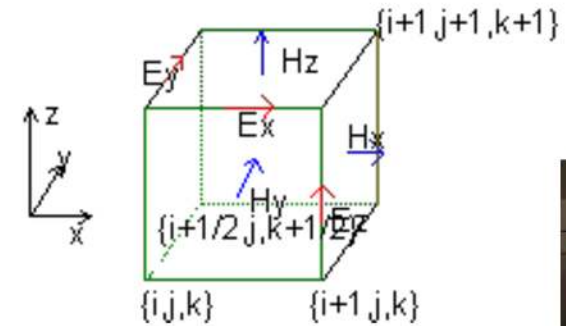
## Memory

- 2 mm resolution
  - $1000 \times 250 \times 250 = 625 \cdot 10^3$  voxels
  - x 6 field components =  $3.75 \cdot 10^3$  data
  - x 8 bytes = 3 GB
  
  - x 4 matrix = 12 GB
  - + boundary = 15 GB

1 millimeter resolution → 120GB



Divide the resolution by 2  
increase the memory by 8 and  
the time computation by 16



## Computation Time

$dt \approx 10^{-12}$  s,  $N_{\text{step}} \approx 1,5 \cdot 10^4$

Speed MPI < 40 M cell/s  
Speed GPU [40 280] M c/s

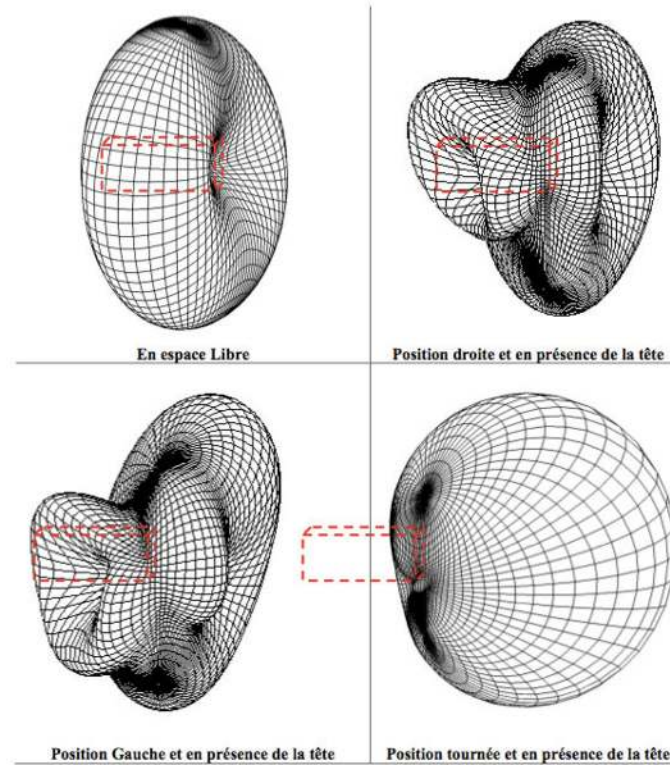
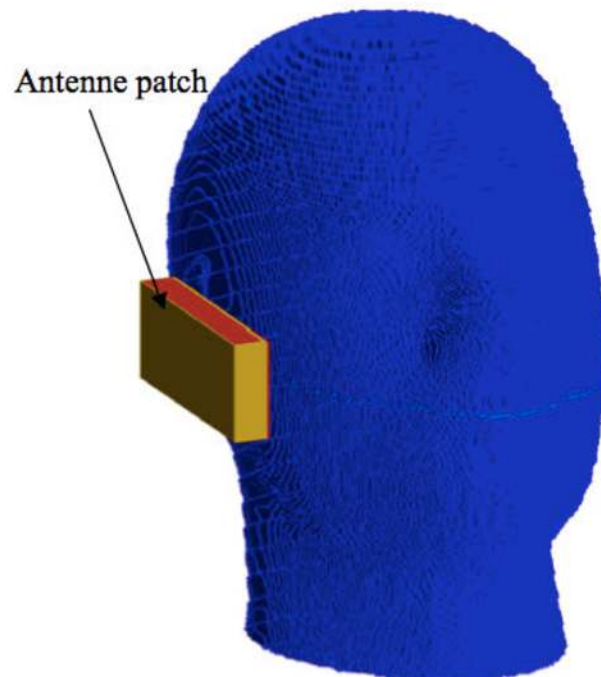
System : 2 CPU intel XEON 12 core 2.4 GHz;  
144 Go ; 4 GPU Nvidia Tesla M2090

Problem:  $440 \times 460 \times 770$  Voxels and  
17030 time steps

**Duration : 2h37 with 280 Mcells/s**



# Example of application: Pattern antenna of a mobile close to the head

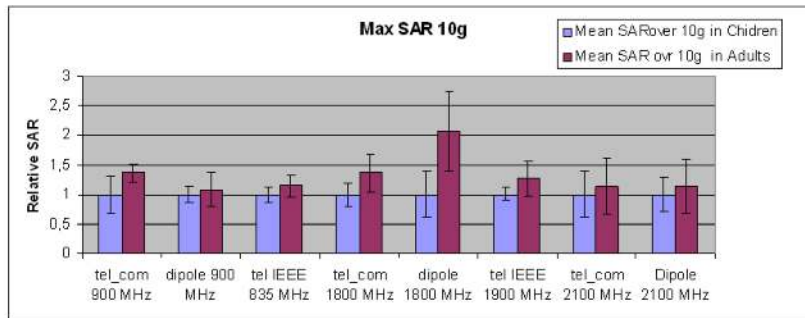
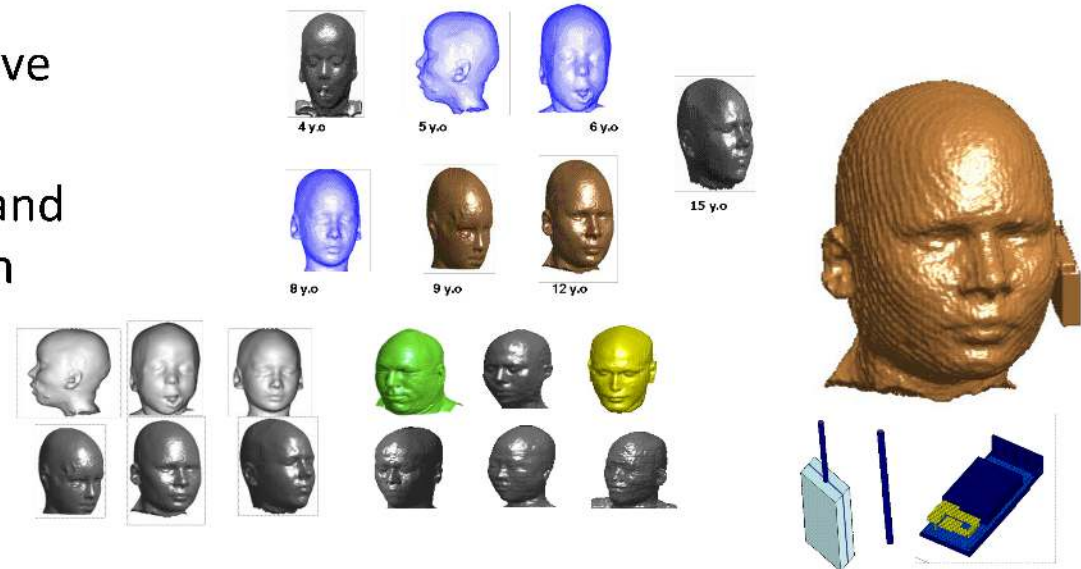




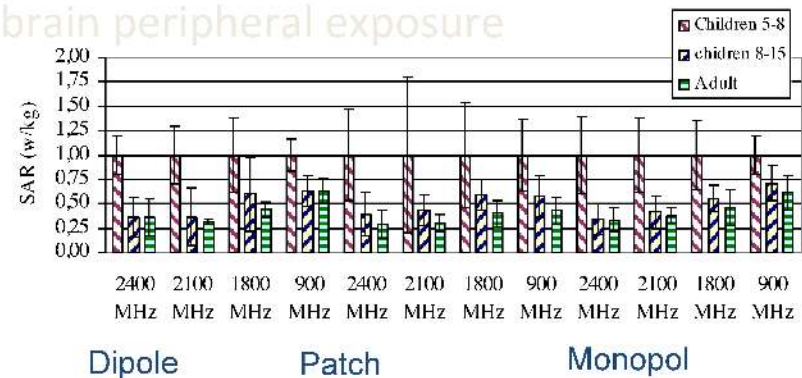


# Children vs Adults exposure

- Head models, MRI based, have been developed
- Comparison between adult and child head models have been conducted .



Locally the exposure can differ:  
e.g brain peripheral exposure

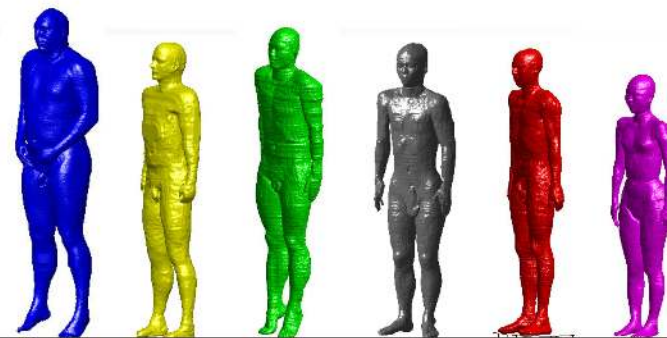
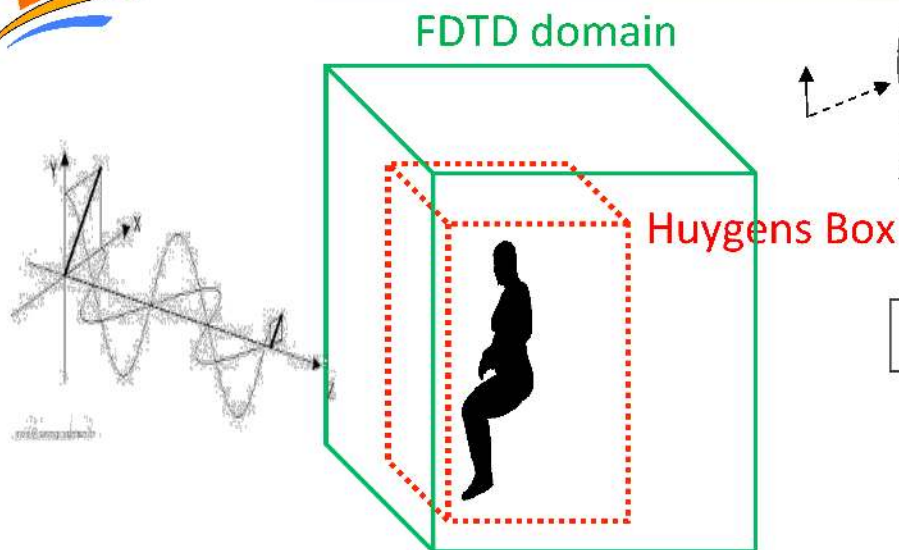


Analysis of RF exposure in the head tissues of children and adults  
J. Wiart, A Hadjem, M F Wong and I Bloch, Phys. Med. Biol. 53 (2008) 3681–3695

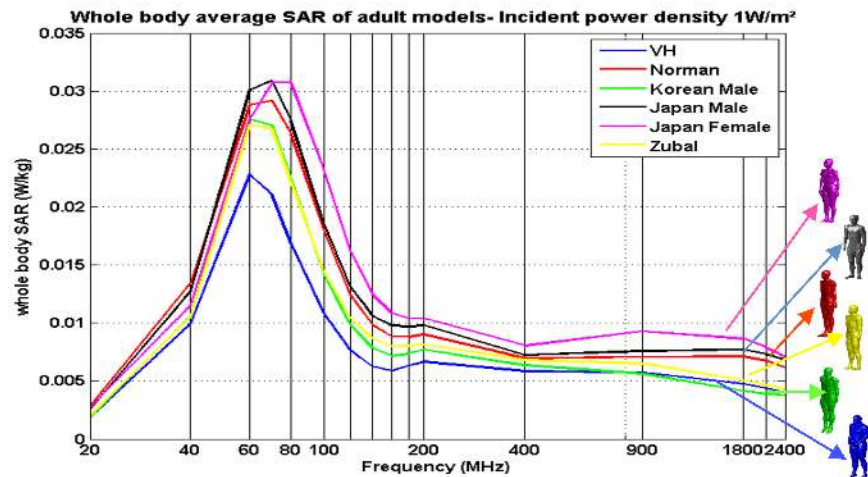
Works performed by N Varsier, A Hadjem E Conil and J Wiart



# Whole Body exposure from a far source



Absorbed power divided by the weight vs frequency



A single simulation (1 phantom, 1 freq) = 8 h  
16 freq x 6 phantoms → 768 hours... 32 days...

Large influence of the morphology

- ### Huygens Box Principle
- The exposure induced by the incident field can be performed using the equivalent principle
  - With the E.P. only the incident field at the surface is required to assess the field inside the box.
  - The far field of base station antenna can be approximated using a plane wave that can be used as incident field





# Human exposure induced by a "femto cell" at home

In this case

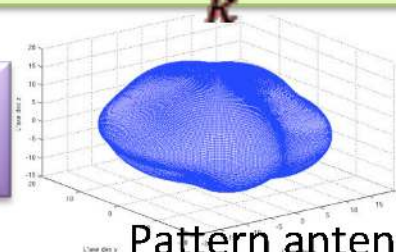
- crude FDTD simulation unrealizable: too big FDTD domain.
- Plane wave approximation is not valide

The field radiated by a source can be expanded over spherical waves (that are an orthogonal base as the plane waves are)

$$\vec{E}(r, \theta, \varphi) = \sum_k Q_k \vec{F}_k(r, \theta, \varphi)$$

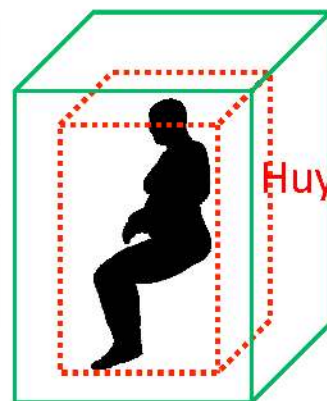


Measurements can be provided the coefficients



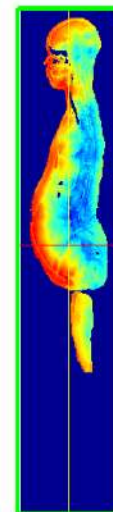
Pattern antenna of the « femto »

Works performed by P. Kersaudy



FDTD domain

Huygens Box



Exposure induced by the femto

*Phd works performed by P Kersaudy  
Supervisors O Picon, S Mostarshedi, B Sudret and J Wiart*

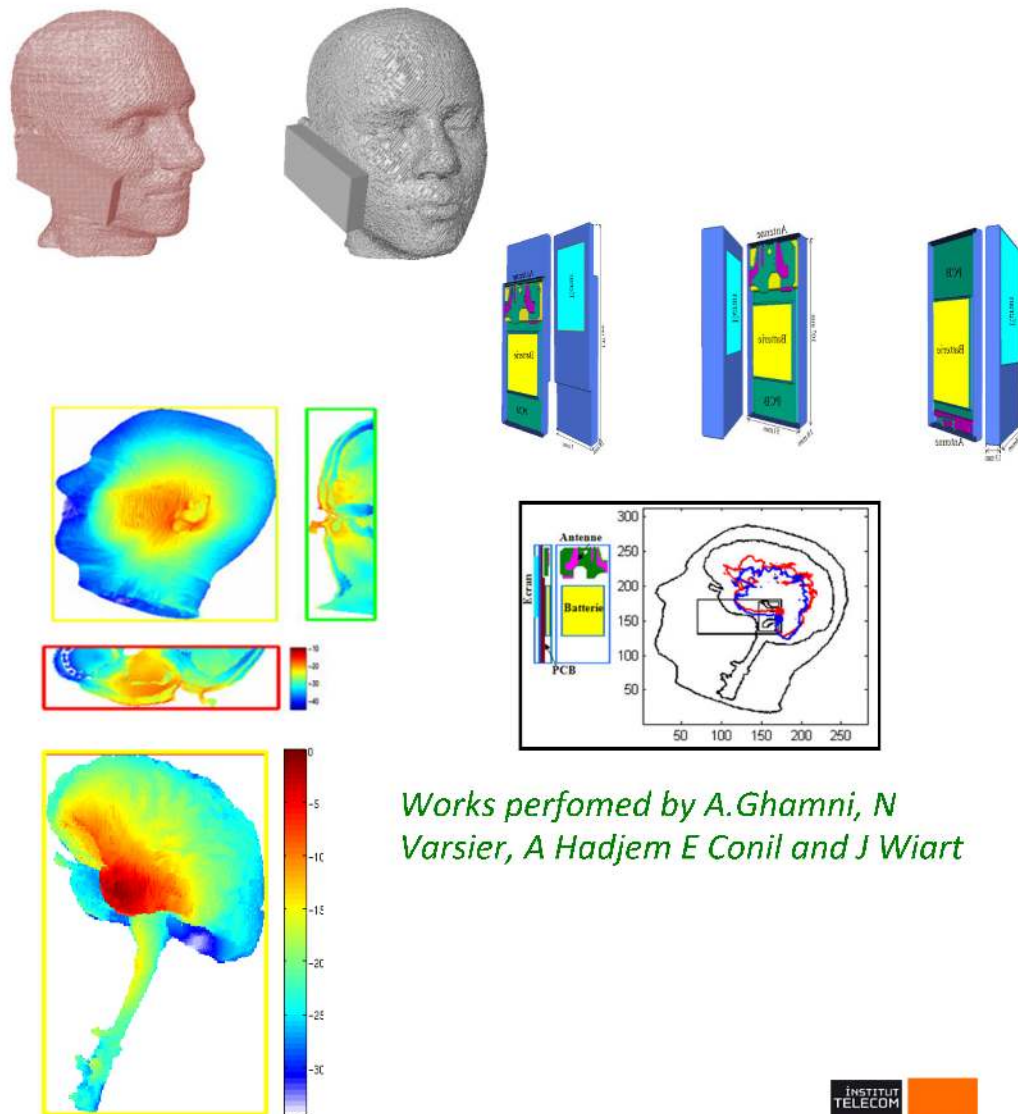




# Human exposure induced by a mobile close to the head

- The source is very close to the tissues. So "Huygens box" cannot be used and must be modelled
- The head models exist.
- The main problem is to locate the phone close to the head
- The accuracy of the simulation depends on the accuracy or the representativeness of the source models

Simulations « head only » request less memory and less time computation (less than 20 minutes)  
But preparation time can be relatively important

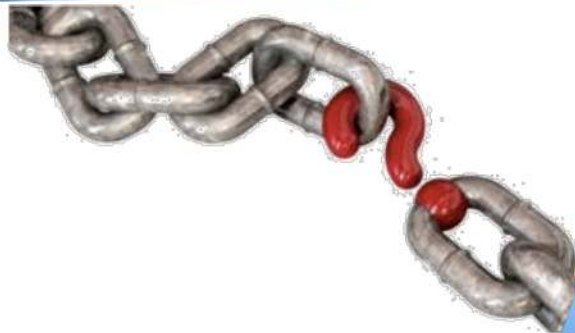


*Works performed by A.Ghamni, N Varsier, A Hadjem E Conil and J Wiart*



# Today exposure is facing variability

DETERMINISTIC  
DOSIMETRY



STOCHASTIC  
DOSIMETRY

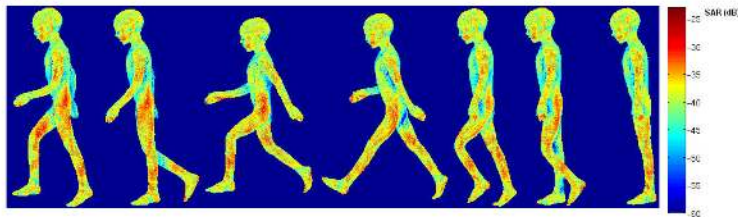
Multiple network technologies

Versatile use of wireless communication

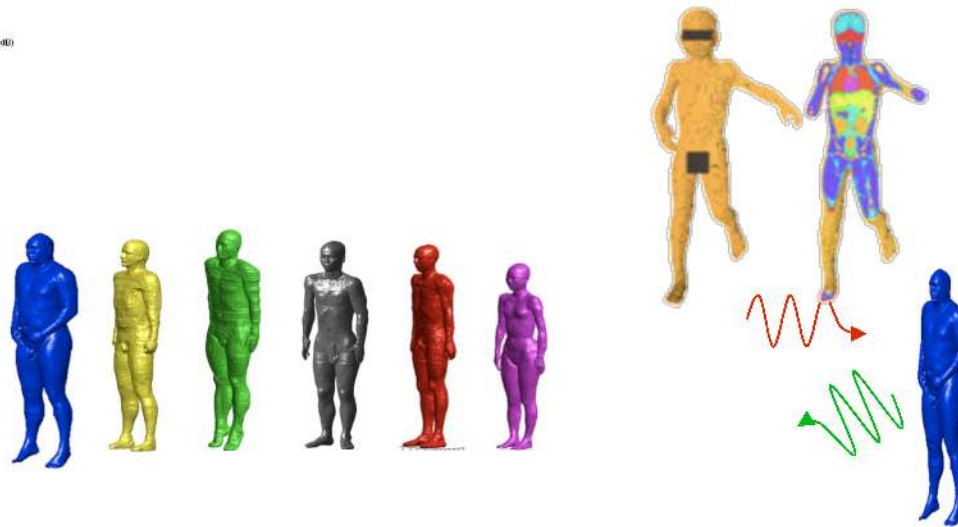
Versatile use of frequencies with  
HETNET and SON



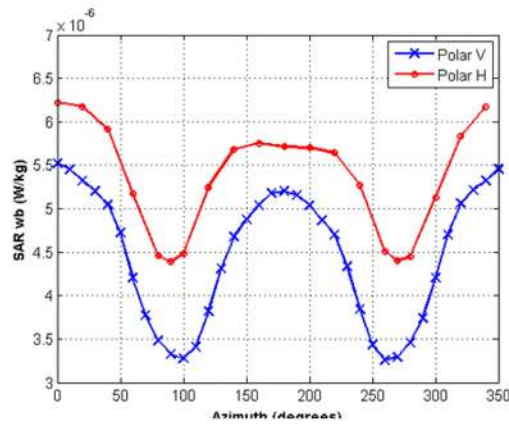
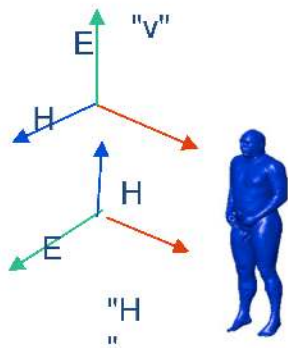
# Morphology, Sources, Posture... have large influence.



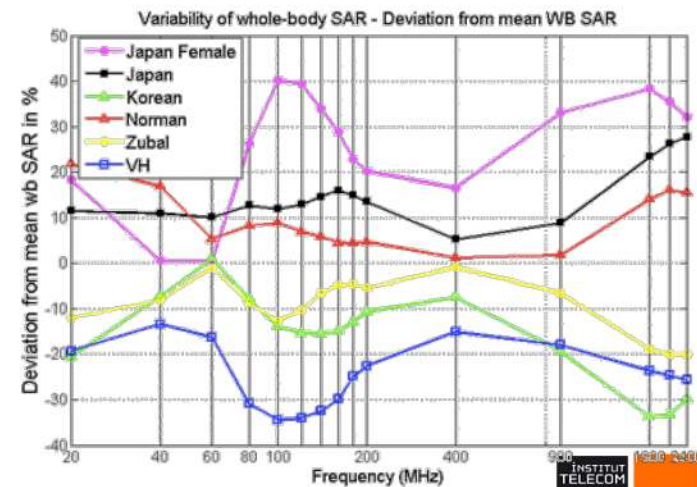
Total absorbed power divided by the weight



## Large Variability



2.1 GHz





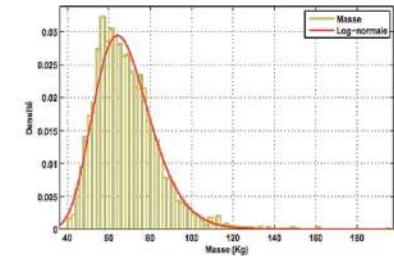
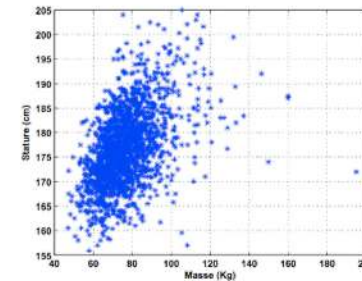
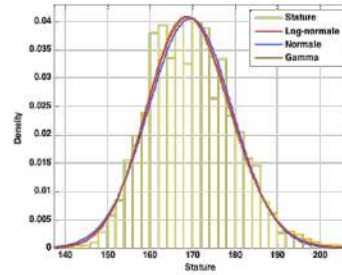
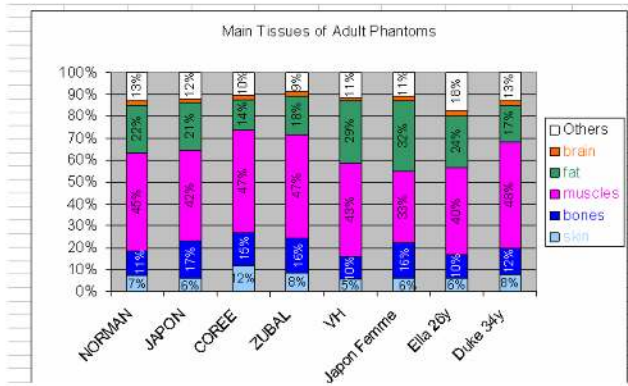
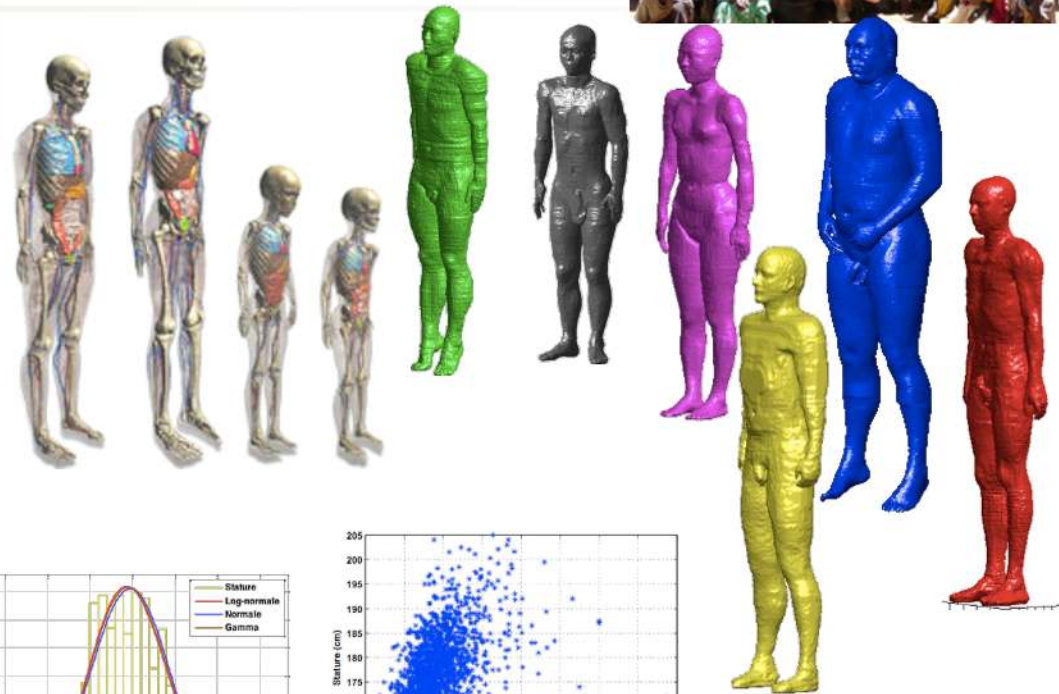


# Variable Morphology

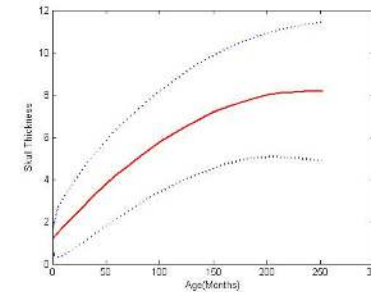
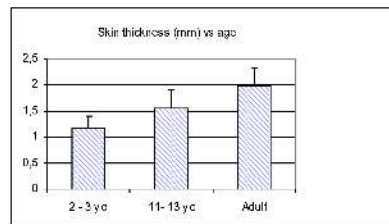


The first challenge has been to get information

Variation within the same age



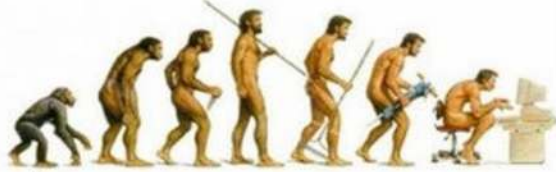
Variation with age





# Evolving technologies and versatile use

Source location and user posture



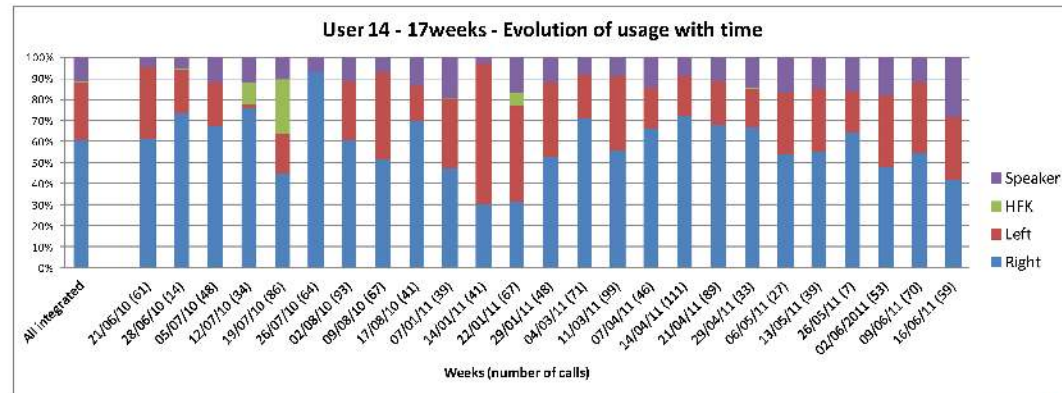
Size and antenna location



Variation of the laterality use



Important for epi studies.

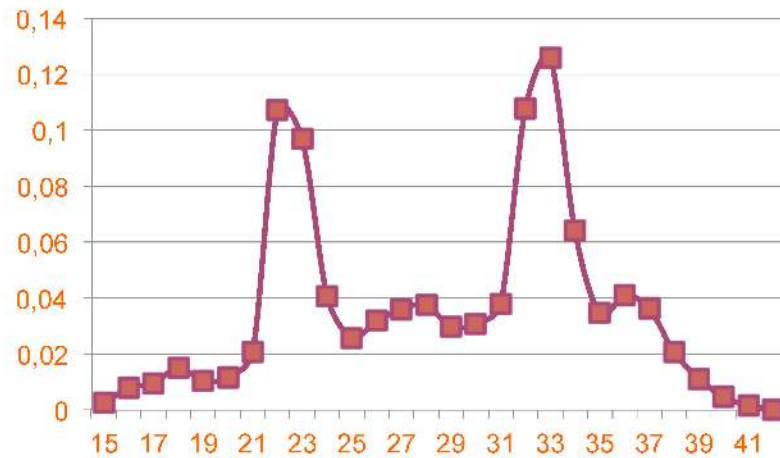




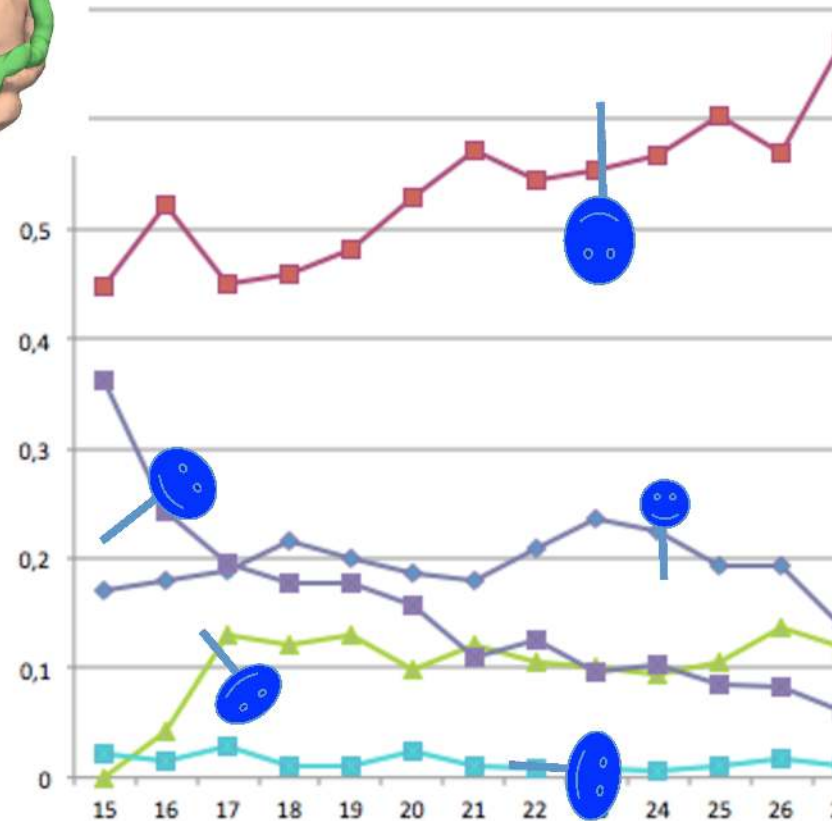
# Variable Posture: e.g Fetus posture



### Observation vs weeks



More than 15000 observations performed at Maternité Port Royal under the responsibility of Docteur Gilles Grangé



Posture occurrence vs weeks of pregnancy





# Dielectric properties uncertainties

- Most of the Dielectric properties measurement have been performed with mamalian animals
- The dielectric properties are tissues and frequency dependent

Standard deviation of the conductivity of the skin of dry face can be 10%

- The dielectric properties of tissues are age dependent
  - A Peyman et al pmb 2009,

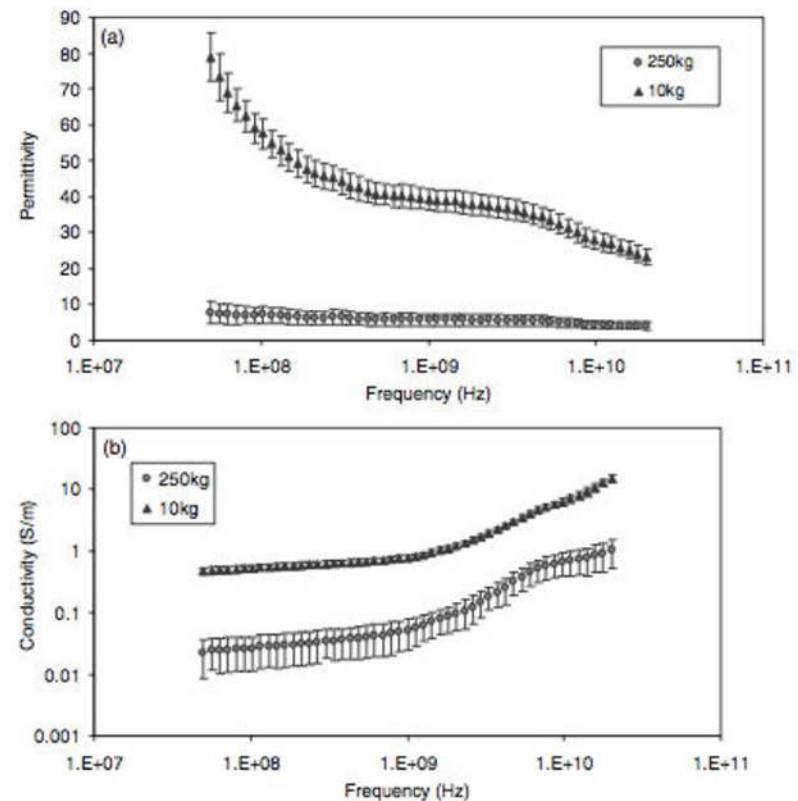
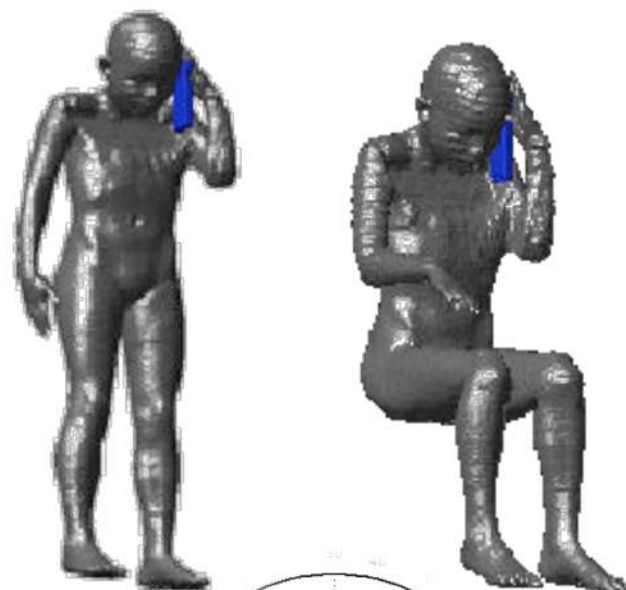


Figure 4. The measured (a) permittivity and (b) conductivity of bone marrow 30% for 10 and 250 kg pigs. The error bars are the total combined uncertainty with  $k = 3$  which represent the 99% confidence interval.



# Variable SAR induced by the variable gain of the couple mobile+user

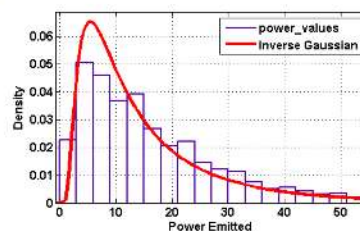
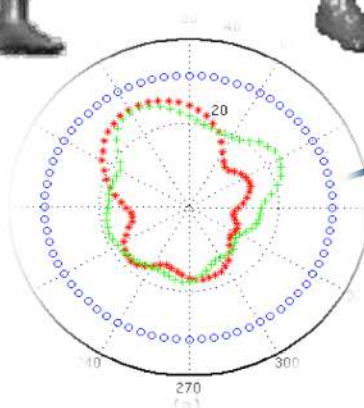


- The pattern antenna (gain) of « mobile +user » depends on the posture and location of the phone
- The human exposure (SAR) depends on the power radiated by the phone



$$P_e * G_e * \underbrace{PL * G_r}_{\text{constant}} = \underbrace{P_r}_{\text{fixed}}$$

$$P_e * G_e(\theta_{LOS}, \phi_{LOS}) = 1$$



Considering PE x GE known, what is the variation of SAR. In this case is the relative position of the phone is fixed so SAR depend only the power emitted:

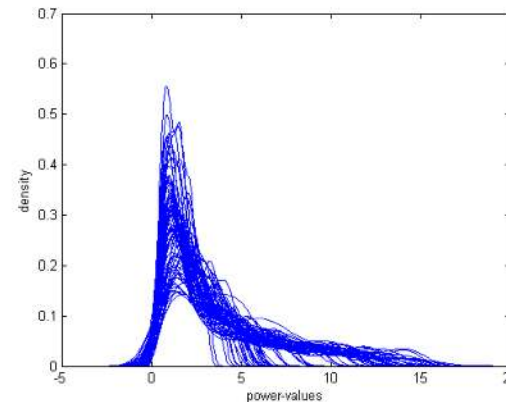
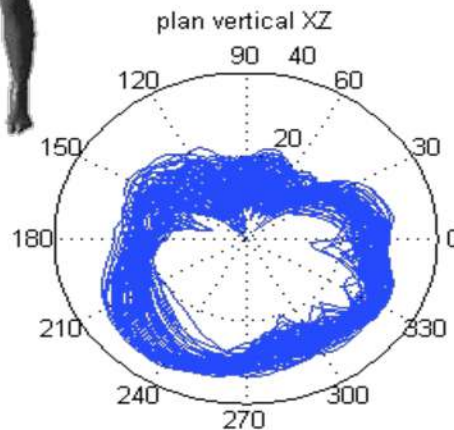
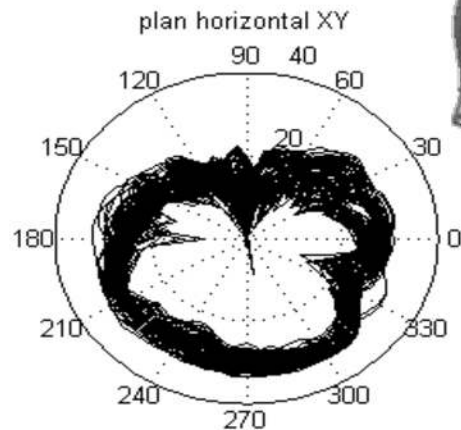
$$P_e = \frac{1}{G_e(\theta_{LOS}, \phi_{LOS})}$$

Phd works performed by A Krayni  
Supervisors A Sibille, A Hadjem and J Wiart



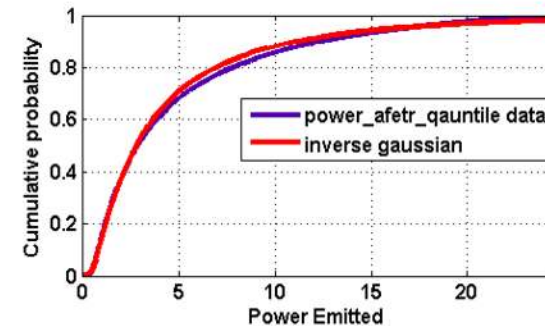
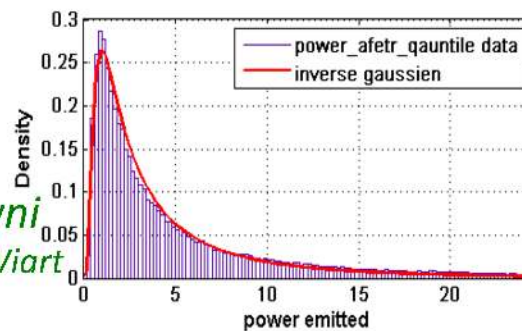
# Variation of the power emitted by a Tablet

3 variables. Y,Z,Teta  
LHS Planning experiment  
80 simulations



Phd Works performed  
by A. Krayni

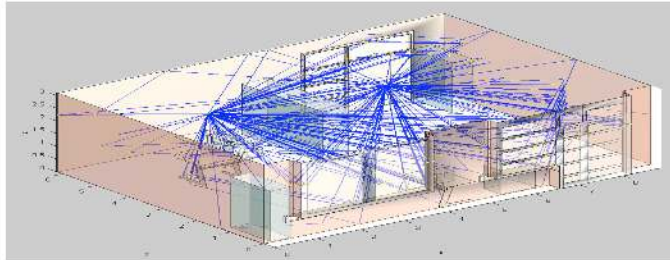
*Phd works performed by A Krayni  
Supervisors A Sibille, A Hadjem and J Wiart*





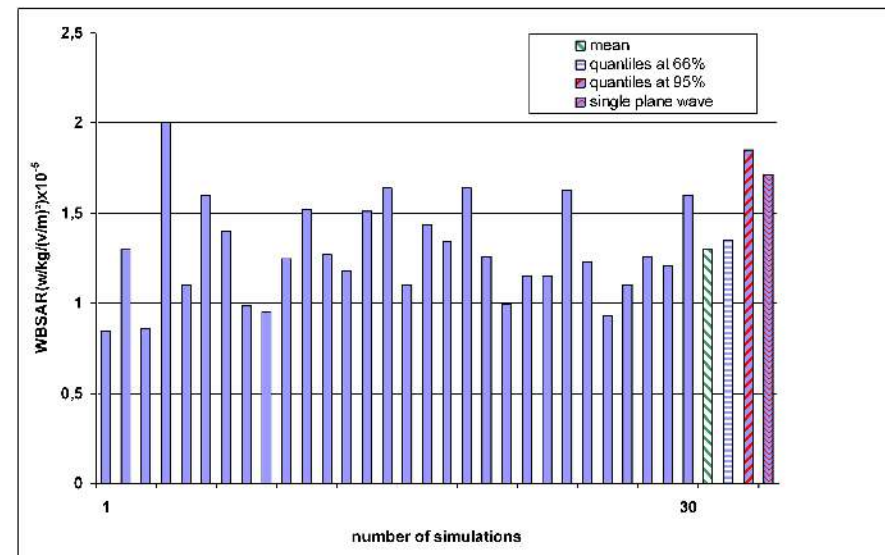
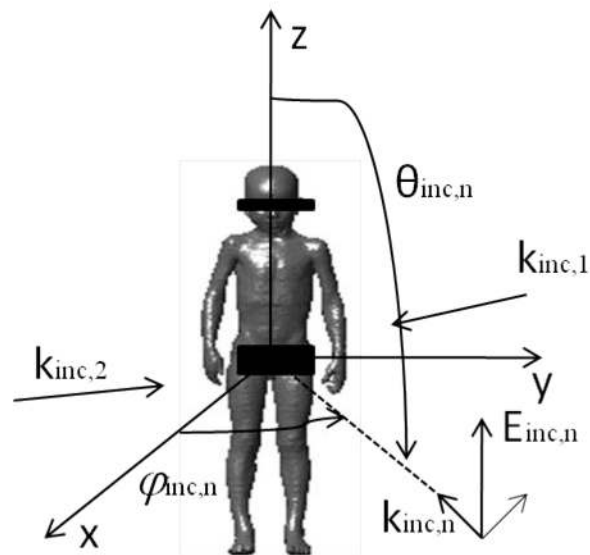


# Variable exposure induced by multiple reflexions



5 incident waves

- Uniform incident angles
- Log normal amplitudes
- Uniform phase



Phd works performed by Th Kientega  
Supervisors O Picon and J Wiart

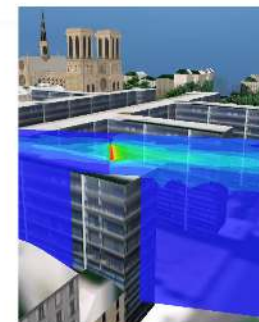
# EMF exposure and Geostatistical tools



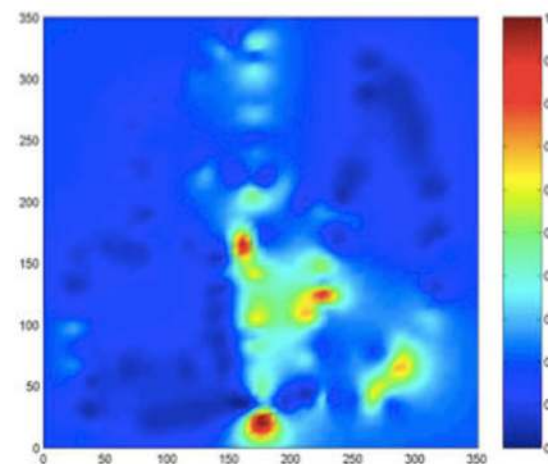
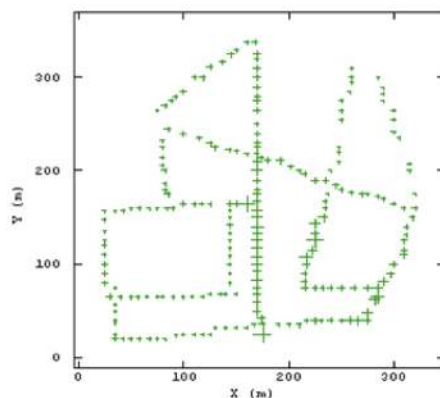
Public questions request exposure assessment

Simulation are facing limitations (e.g. dielectric properties, geometrical characteristic of the buildings, simulation time...)

Measurement are also facing limitations (e.g. uncertainty of the assessment, fading, number of measurements..)



Ordinary Kriging applied

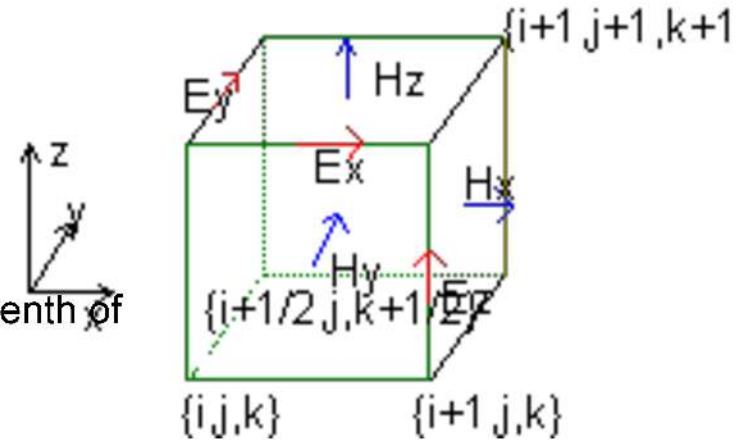


*Phd works performed by Y Ould Isselmou  
Supervisors : H Vandernackel, W tabbara and J Wiart*

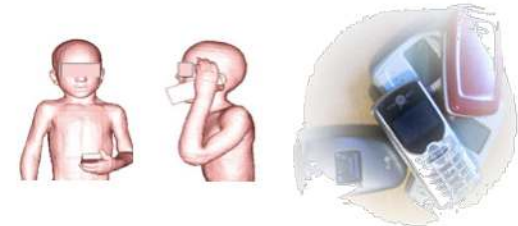


# Variability management: the constraints

- FDTD calculations
  - Whole body : few hours
  - Head only : few tenth of minutes
- Simulation preparation
  - Model deformation and source positioning : from tenth of minutes to few hours
- Human model
  - Limited number.
  - Model Development : from few month to few years
- Source models
  - Model Development : few weeks to few months



Surrogate model are needed



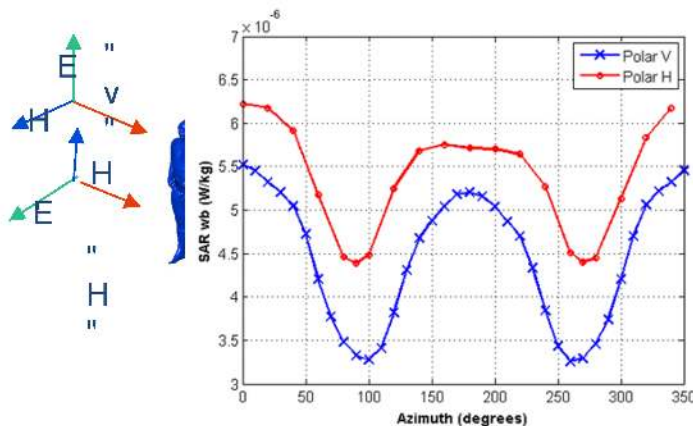
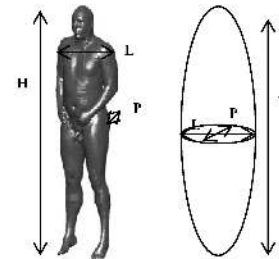




# Simplistic Approach of the whole body exposure

- In the RF domain there is no resonance, then the whole body SAR is proportional to the surface cross section
- Human body can be approximated using Ellipsoid

$$surface = \pi \frac{H}{2} \sqrt{\frac{L^2}{2} \cos(\theta)^2 + \frac{P^2}{2} \sin(\theta)^2}$$



Based on simulations performed with VH

$$P(W) = 0.72 * S(m^2) * DSP \left( \frac{W}{m^2} \right) \pm 5\%$$



Representativeness?

Works performed by E Conil, A Hadjem MF Wong and J Wiart



# Specificity of Dosimetric Problem.

**++ SAR is a regular function**

**-- simulation cost does not allow large number of sampling**

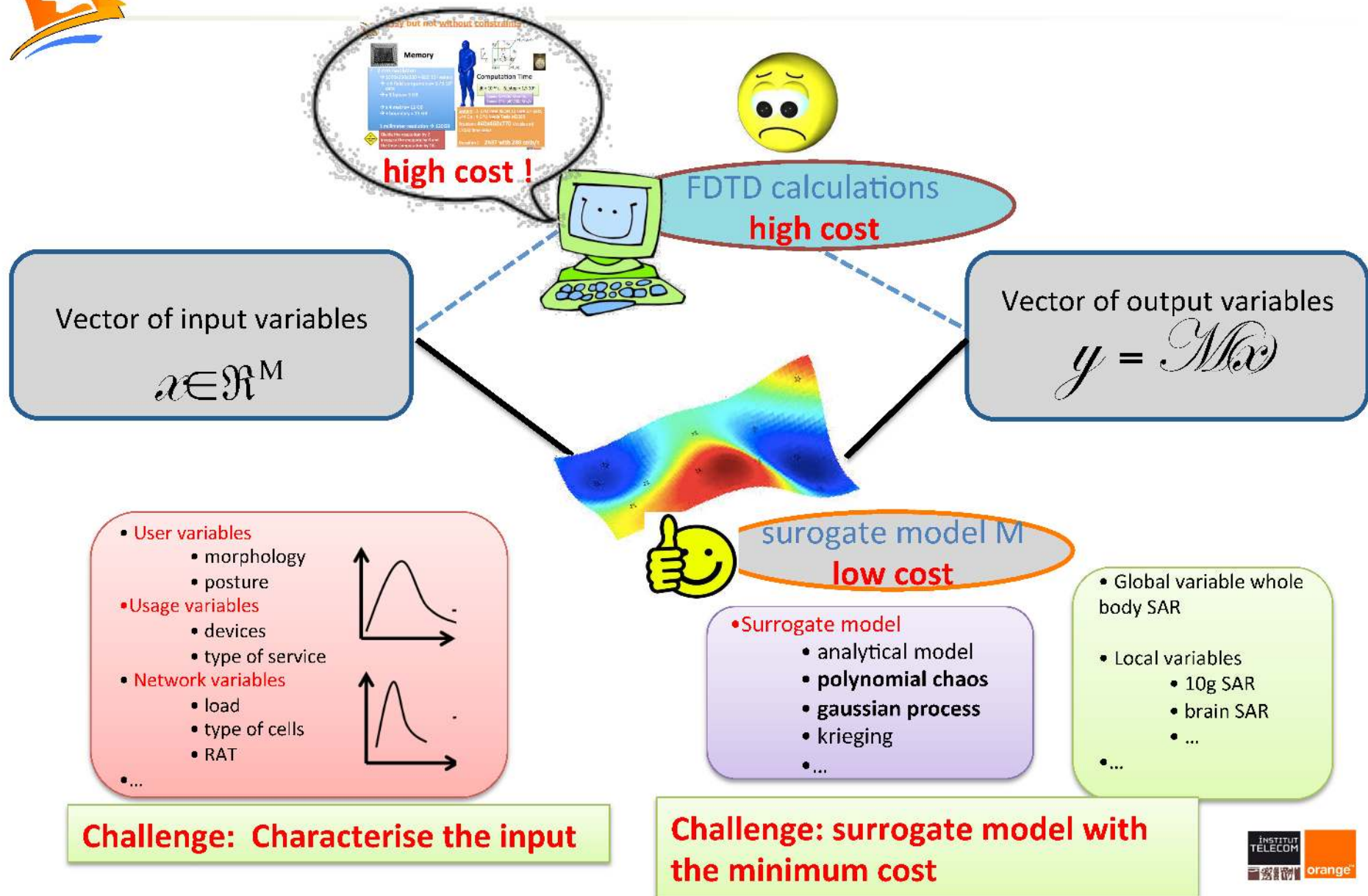
**large number of input**

**small number of human model**

**simulation preparation can be heavy**

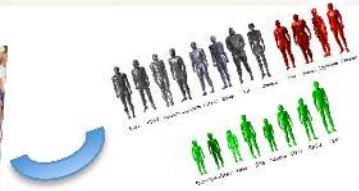
**Simulations are very heavy and long to carry**

# Challenges of the stochastic dosimetry

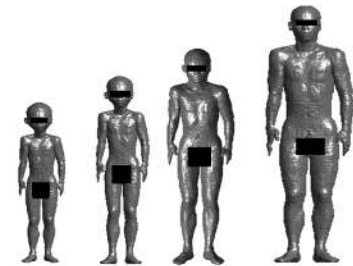




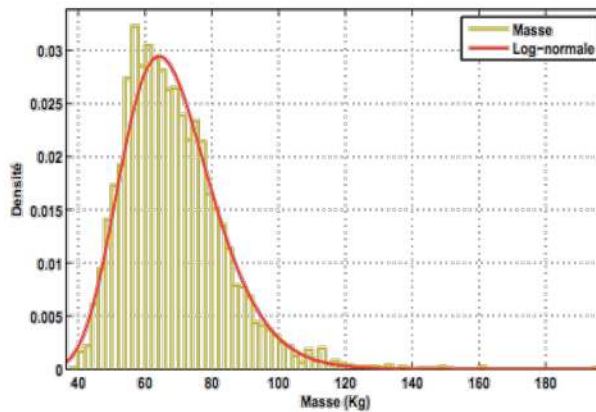
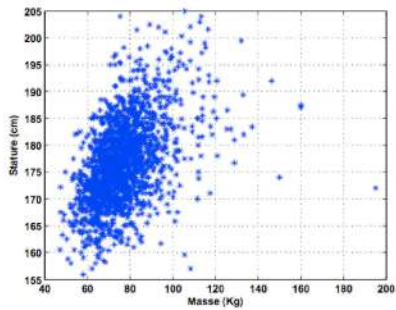
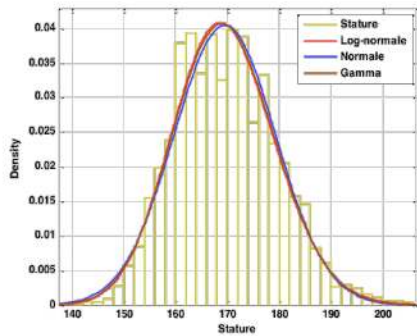
# Morphology and exposure how to deal with?



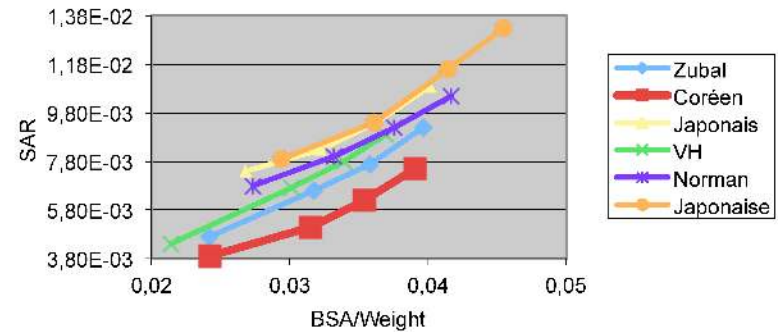
Morphing can be performed based on external data



3D models are missing  
Some data are known by



SARwb vs BSA/Weight



# Whole Body SAR vs morphology first approach



$$y_{WBSAR,1} = \alpha x_{bsa} + \varepsilon_1$$

$$y_{WBSAR,2} = \beta x_{bmi} + \varepsilon_2$$

$$y_{WBSAR,3} = \gamma x_{poids} + \varepsilon_3$$

- $Y_{WBSAR,i}$  : Whole body SAR WBSAR
- $x_{bsa}$  : Body Surface Area/Weight
- $x_{bmi}$  : (BMI)<sup>-1</sup>
- $x_{poids}$  : (Weight)<sup>-1/3</sup>
- $\varepsilon_1, \varepsilon_2, \varepsilon_3$  : models errors

**WBSAR model :  $Y = \beta \cdot X$**   
Where  $X = \text{BMI}^{-1}$

Facts:  $\beta$  is positive, there is an upper limit

Additional hypothesis:

- The existing phantom have been built to represent mean population

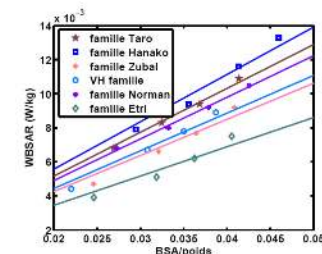
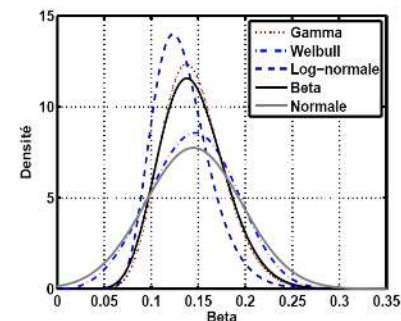
$$\langle \beta \rangle = 0.15$$

- $\beta$  min = 0
- symmetry of  $\beta$   $\beta$  max = 0.3

Using morphing

Mean square regression Based on 12 body models

Model	Param. estimé	p-valeur t-test	I.C à 95%	R <sup>2</sup>
BSA/weight	0.25	10 <sup>-11</sup>	[0.21,0.28]	0.78
BMI <sup>-1</sup>	<b>0.15</b>	10 <sup>-11</sup>	<b>[0.14,0.16]</b>	<b>0.87</b>
weight <sup>-1/3</sup>	0.027	10 <sup>-6</sup>	[0.0235,0.03]	0.67

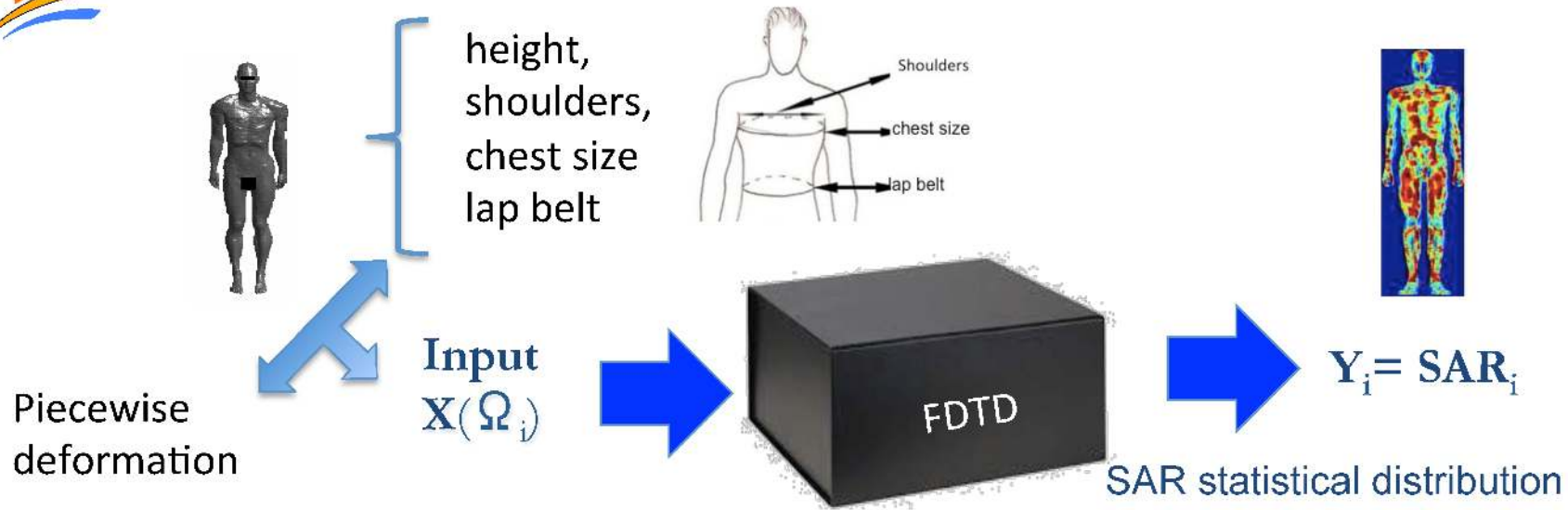


law	95% quantile WBSAR (mW/Kg)
Gamma	10
Beta	9.8
Normal	11
Weibull	10.5
Log-normal	9.9

Phd works performed by A El Habachi  
Supervisors E Vasquez, G Fleury and J Wiart



# Second approach using Polynomial Chaos



$$Y = M(X) \quad \text{With} \quad E(Y^2) < \infty$$

$$Y = \sum_k \beta_k \Psi_k(X)$$

Where  $\beta_k$  are the coefficients of the polynomial chaos expansion

$\Psi_k$  are the basis of of the polynomial chaos.





# Projection and Quadrature approach to get the coef

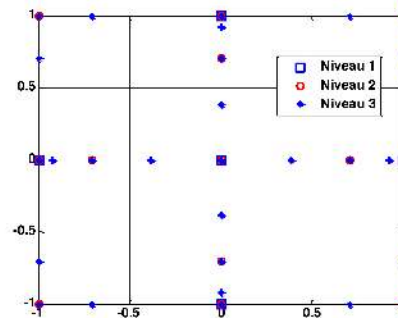
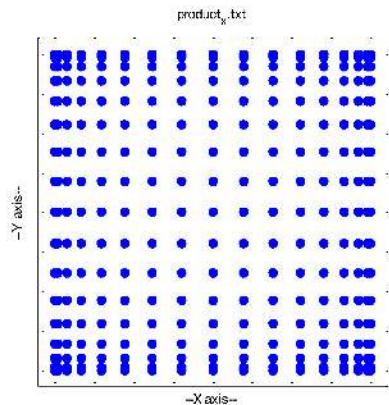
$\Psi_k$  are orthogonal

Modal description is often used in electromagnetism and therefore the projection can be considered as "natural" in dosimetry.

$$\beta_k = \int Y(x) \Psi_k(x) f_X(x) dx$$

$$\beta_k = \frac{1}{\|\Psi_k\|^2} \int SAR(x) \cdot \Psi_k(x) pdf_X(x) dx.$$

In fact even with quadrature, the projection approach leads to have large number of FDTD simulations.



	3 D	4 D
Ordre 1	7	9
Ordre 2	25	41
Ordre 3	69	137
Ordre 4	177	401
Ordre 5	441	1105

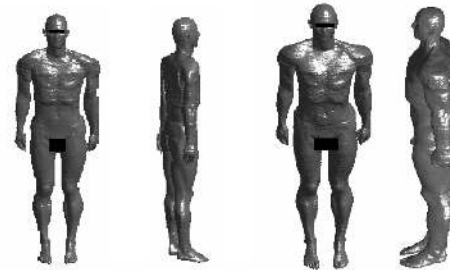
*Phd works performed by J Silly Carette  
Supervisors V Fouad Hanna, D Lautru MF Wong and J Wiart*

**Variability on the Propagation of a Plane Wave Using Stochastic Collocation Methods in a Bio Electromagnetic Application**  
Silly-Carette et al IEEE MWCL 2009

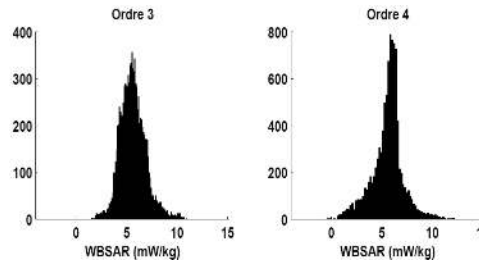
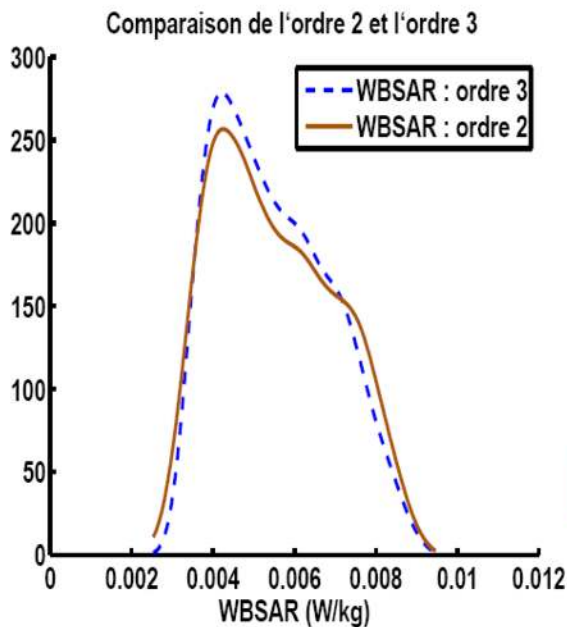


# WBSAR vs morphology using Polynomial Chaos and projection

- Sparse Quadrature : « Clenshaw Curtis »
- Smolyak tensorisation
- 4 input parameters: height, shoulders, chest size, lap belt
- Morphed human models



Simulations cost  
 ~ 17 days with GPU, 4 months CPU



	3 D	4 D
Ordre 1	7	9
Ordre 2	25	41
Ordre 3	69	137
Ordre 4	177	401
Ordre 5	441	1105

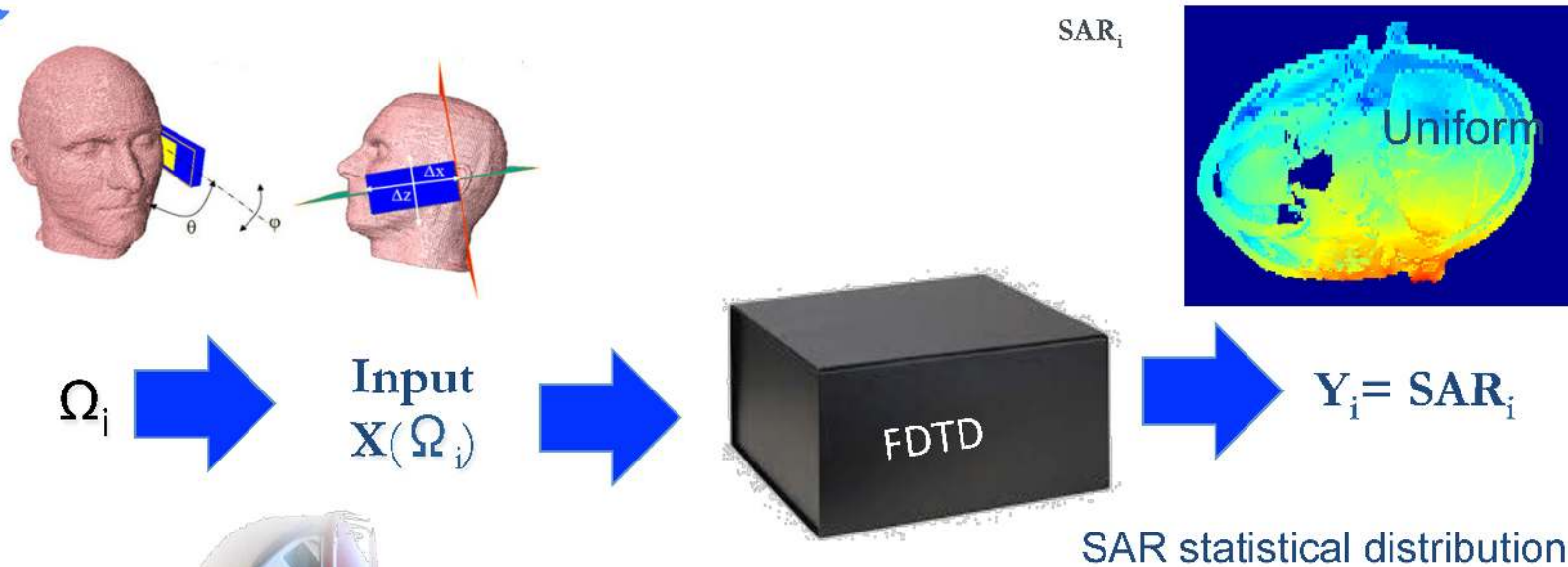
Mean = 5.3 mW/kg  
 95% quantile WBSAR = **7.9 mW/kg**

The projection approach is not suitable for dosimetric problems

*Phd works performed by A El Habachi  
 Supervisors E Vasquez, G Fleury and J Wiart*



# Influence of the phone position closed to the head using Polynomial Chaos



- same head
- same phone
- different phone position

	$\theta$	$\Phi$	$\Delta x$	$\Delta z$
	[0 -30°]	[-15°+15°]	[5 - 30 mm]	[-10 +10mm]



# Coefficients assessment using Regression



Since the projection approach is not suitable for dosimetric problems we used regression

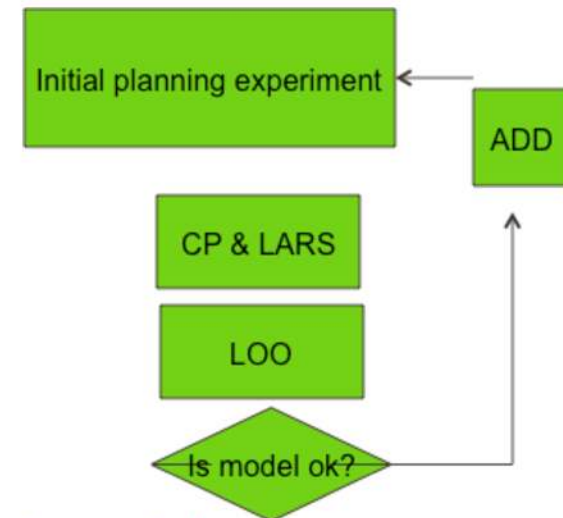
$$\hat{Y} = \sum_{k=1}^N \beta_k \Psi_k(X)$$

Considering a truncature Regression can be used to get the coefficients.

$$\hat{y} = \begin{pmatrix} \hat{y}_0 \\ \hat{y}_1 \\ \vdots \\ \hat{y}_P \end{pmatrix}$$

$$Z = \begin{pmatrix} \Psi_0(\xi^{(1)}) & \Psi_1(\xi^{(1)}) & \dots & \Psi_P(\xi^{(1)}) \\ \Psi_0(\xi^{(2)}) & \Psi_1(\xi^{(2)}) & \dots & \Psi_P(\xi^{(2)}) \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_0(\xi^{(n)}) & \Psi_1(\xi^{(n)}) & \dots & \Psi_P(\xi^{(n)}) \end{pmatrix}$$

$$\hat{\beta} = (Z^T Z)^{-1} Z^T y$$



Leave one out to analyse the global accuracy of such model

*If the model is not as expected then a new experiment has be added*



# Influence of the phone position closed to the head

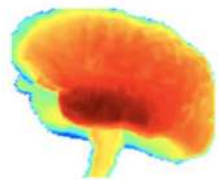
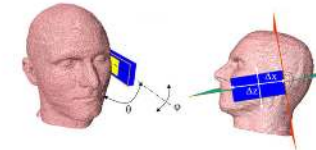
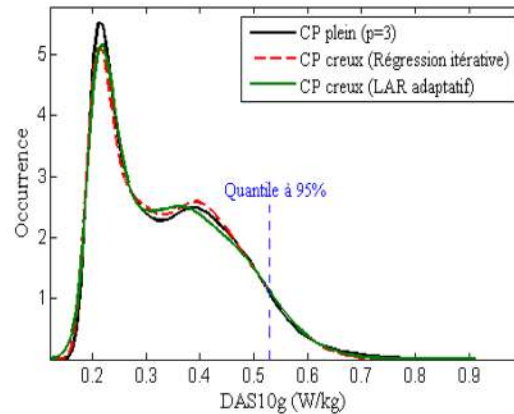
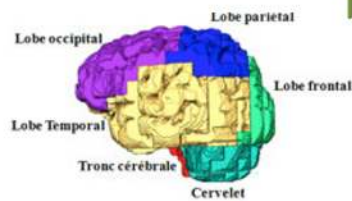
Latin hyper sampling

Uniform

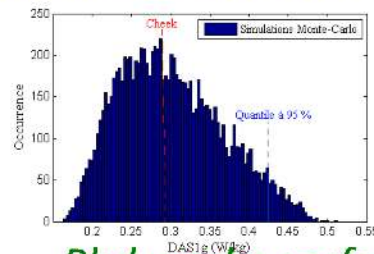
PC + LOO → 122 FDTD simulations

Maximum SAR over 10g in the head

	$\theta$	$\Phi$	$\Delta x$	$\Delta z$
	$[0 - 30^\circ]$	$[-15^\circ + 15^\circ]$	$[5 - 30 \text{ mm}]$	$[-10 + 10 \text{ mm}]$

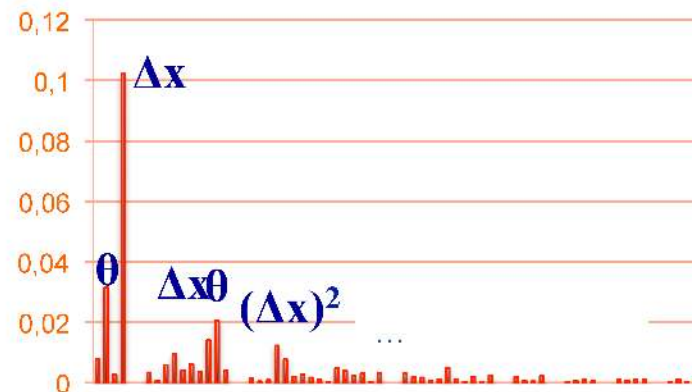


Maximum SAR over 1g in the brain



Phd works performed by A Ghanmi  
Supervisors O Picon and J Wiart

Sensitivity analysis

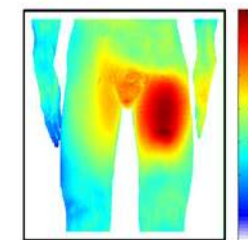
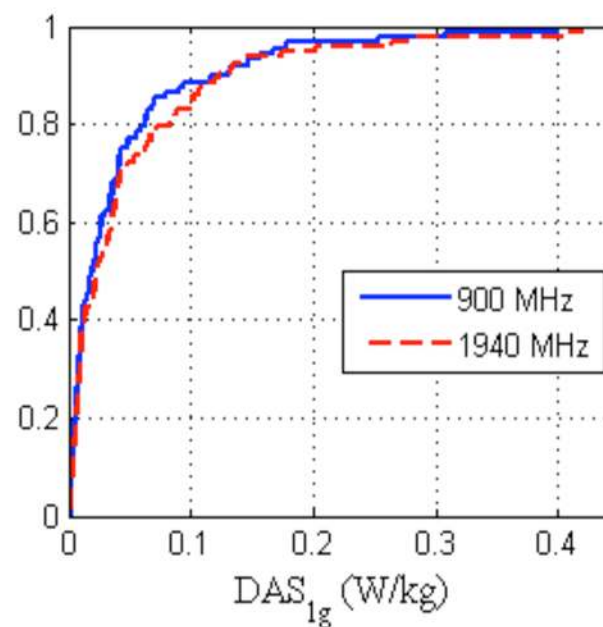
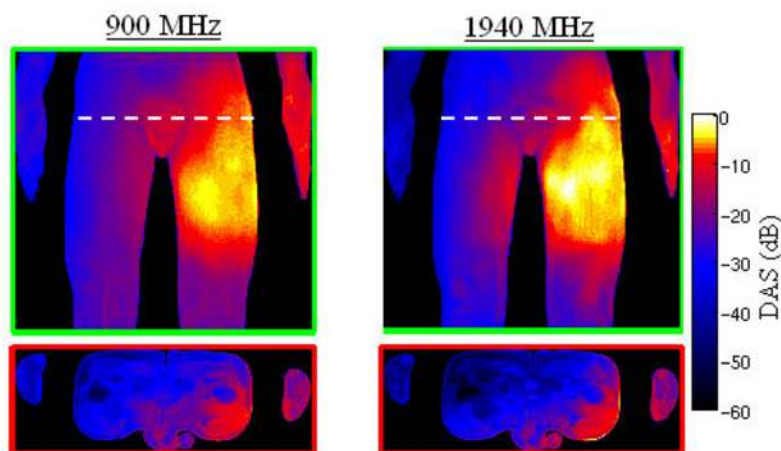
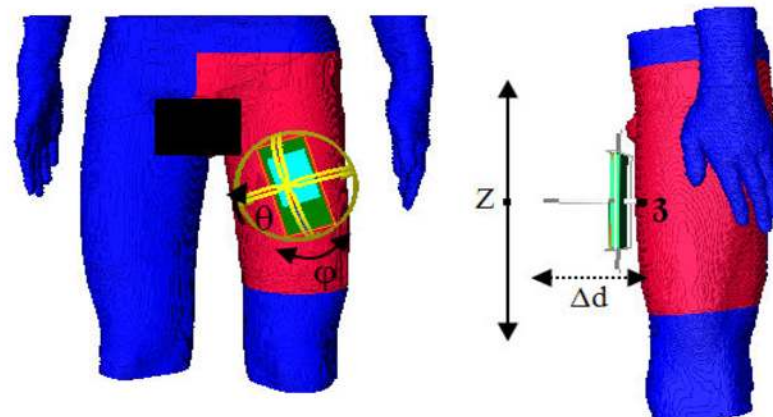




# Genital organs exposure

□ 4 inputx

	Z	$\theta$	$\phi$	$\Delta d$
Intervalle	$\pm 122$	$[-90^\circ 60^\circ]$	$[0 360^\circ]$	$[1 10 \text{ mm}]$

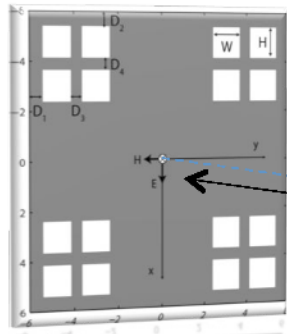


*Phd works performed by A Ghanmi  
Supervisors O Picon and J Wiart*





# Exposure from reflection on wall

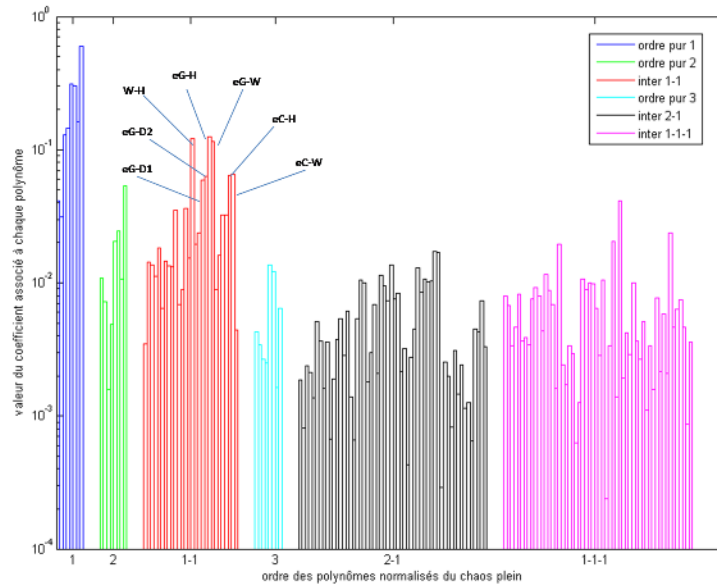


Sensitivity analysis

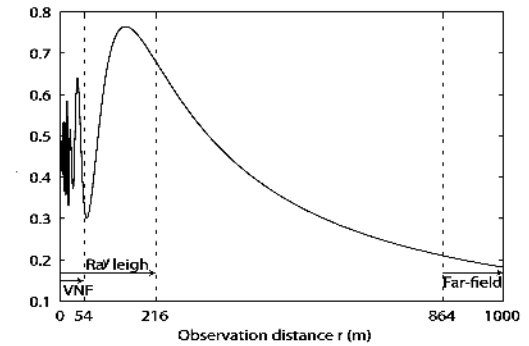
■ 9 inputs

- concrete and glass permittivity
- height and width of windows
- distance between windows
- distance between windows and edges

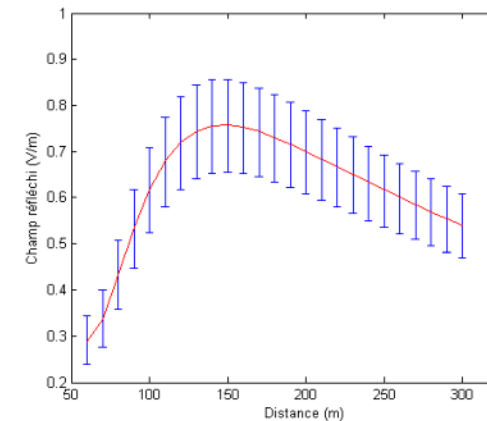
- Physical analysis: observation of the most inflent polynomials
- Prevalence of some interaction terms compared to the corresponding pure order terms



Deterministic approach



With PC

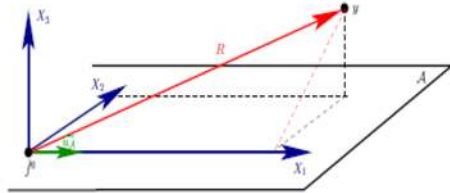
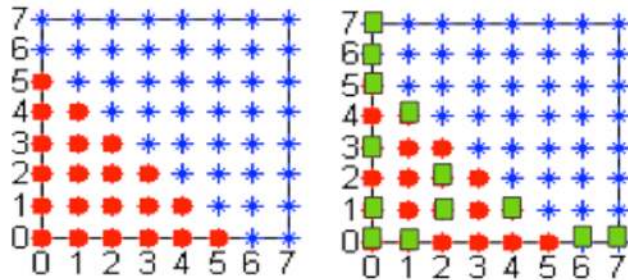


Phd works performed by P Kersaudy  
Supervisors O Picon, S Mostarshedi, B Sudret and J Wiart

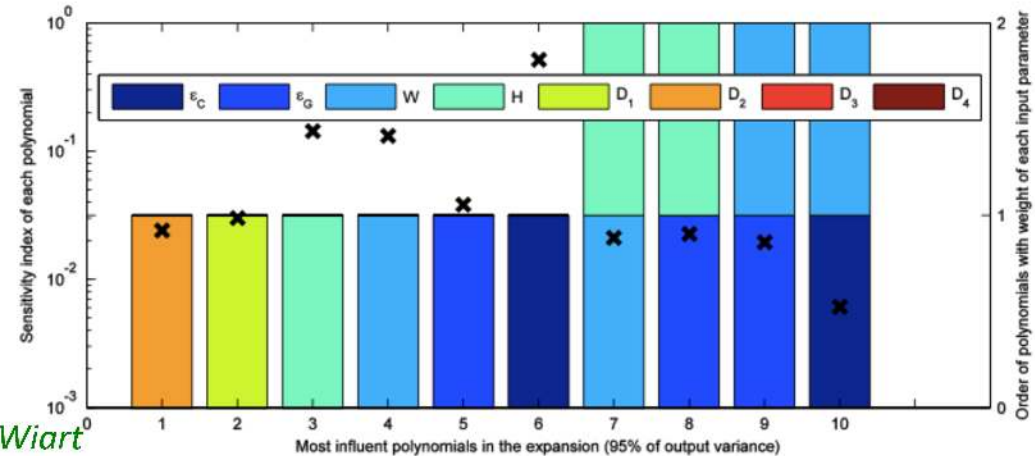
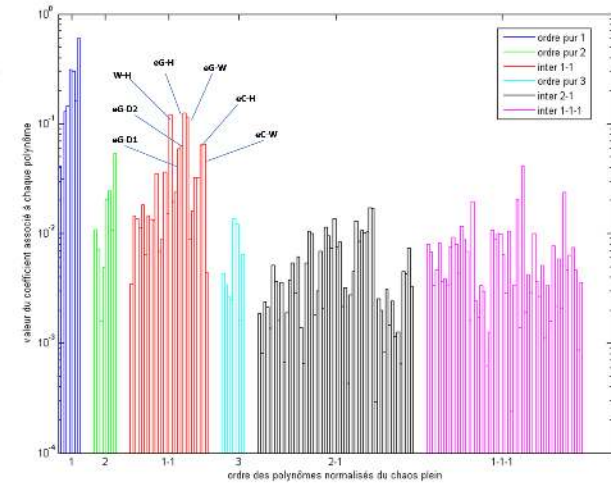


# Limit the computational effort

- As shown in previous analysis all the polynomial are not playing the same role.
- How select relevant polynomials?



Phd works performed by P Kersaudy  
Supervisors O Picon, S Mostarshedi, B Sudret and J Wiart



## Least Angle Regression LARS

Sparse LARS truncation gives a significant reduction of the requested number of simulations

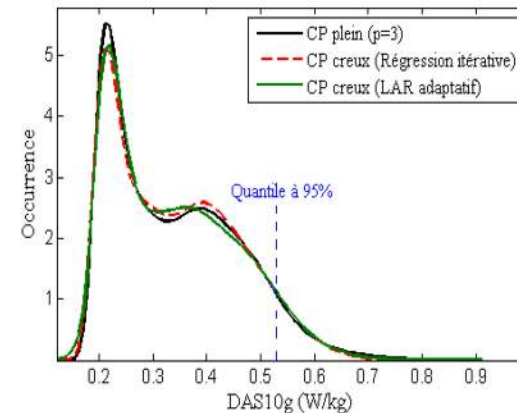


# Next step : parsimonious iterative experiment for quantile estimation

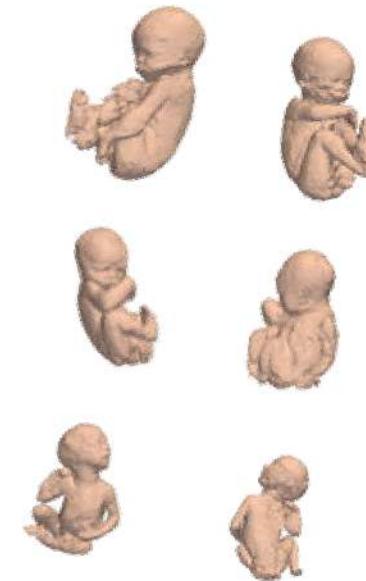
Most of the previous works were dedicated to build a surrogate model able to be used to characterise the shape of the distribution

But the exposure quantification often requests quantile estimation

With the PC and the LOO the uncertainty of the surrogate model is mean square error



A challenge is therefore to build an iterative planning experiment able to monitor the uncertainty of specific quantile



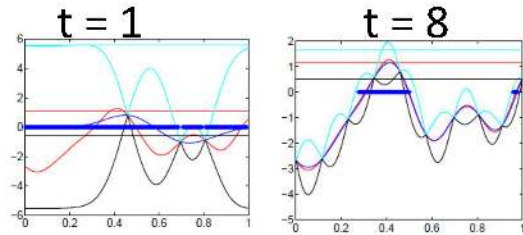




# Fetus exposure Quantile estimation using Gaussian Process Shrunken (GPS)

Phd works performed by M Jala

Supervisors : E Moulines, CLévy-Leduc, E Conil and J Wiart

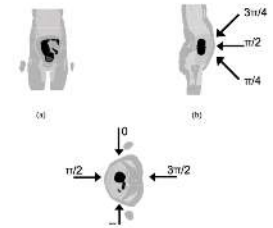
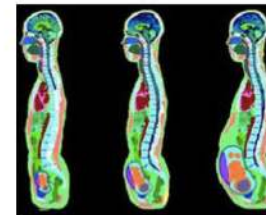


SAR in the brain of the fetus

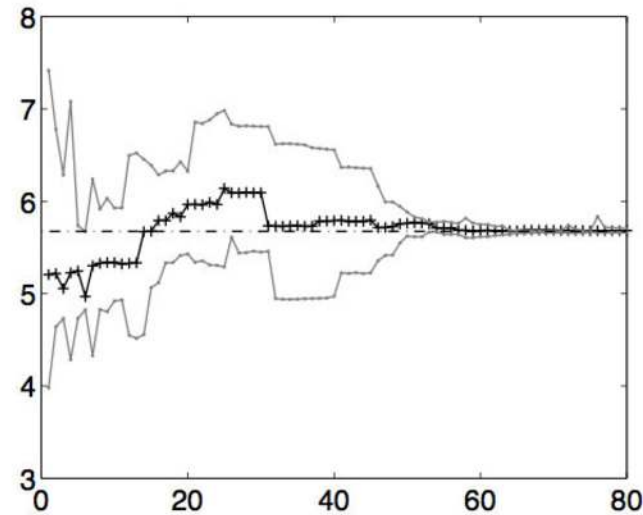
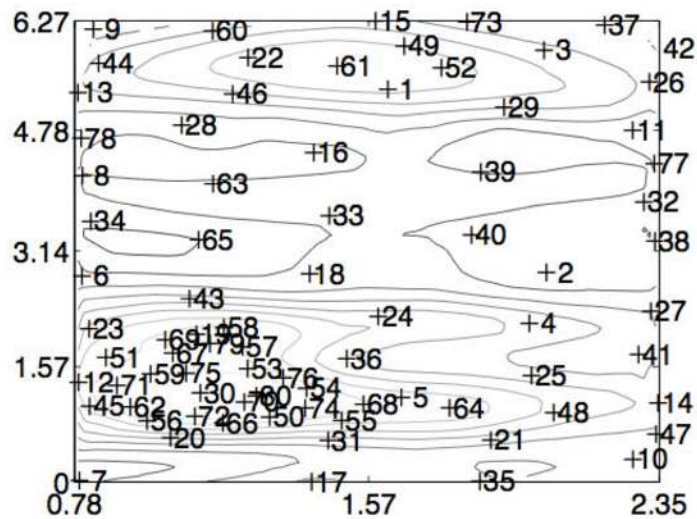
Heterogeneous model

2 input parameters : angles of arrival

GPS with gaussian kernel



Sampling



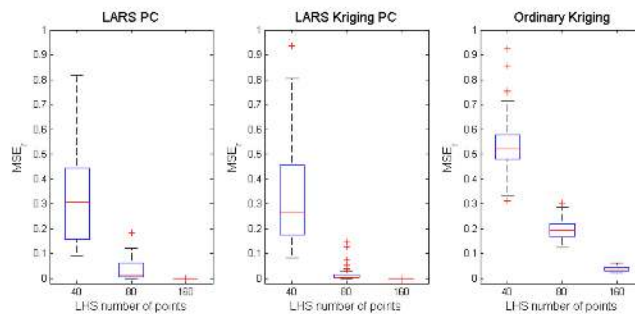
Quantiles vs iteration



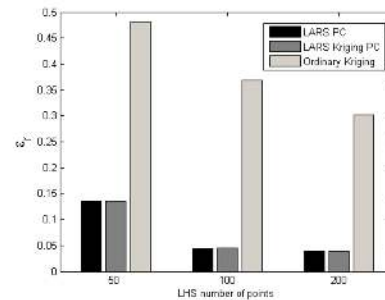
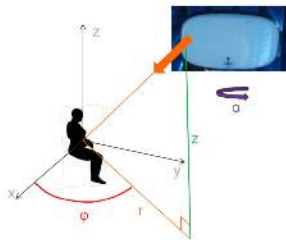
# On going : Combination of Kriging with chaos polynomials

P. Kersaudy poster  
in Mascott Num 2014

Result with Ishigami function



## Application to Fetus exposure induced by femtocell



Phd works performed by P Kersaudy  
Supervisors O Picon, B Sudret and J Wiart

**Use of Chaos Polynomials in a Universal Kriging Model: Application to the Numerical Dosimetry**  
P. Kersaudy<sup>1,2,3</sup>, O. Picon<sup>1</sup>, J. Wiart<sup>1,3</sup>

**Objectives**

- Use polynomials of chaos that are efficient on the output as regression function in the universal kriging model.
- Applying statistical methods to take into account the effect of the input parameters variability on the fetus exposure.

**Context**

- Numerical dosimetry is facing increasingly varied uses.
- The influence of the input parameters variability on the exposure cannot be analysed with usual statistical methods, since the SAR calculation needs high computation time.
- Need advanced statistical methods to assess the SAR distribution.

**Methodology**

- Universal kriging (UK):  $Y(x) = \sum_{k=1}^{p-2} a_k \psi_k(x) + Z(x)$
- Polynomial Chaos (PC) sparse representation using LARS algorithm to select the polynomial in the function:  $Y = \sum_{i \in \text{model}} a_i \psi_i(X)$
- Use of polynomials selected by LARS as regression function in the universal kriging model.
- Adding of relevant information about the output in the kriging model would help to improve it.

**Analytical example: Ishigami function**

**Application: Fetus exposure to a femtocell**

**Input parameters**

- Cylinder coordinates
- Position of the femtocell device on its base

**Output variable**

- Power absorbed by the fetus

**Results**

- Comparison of the performance of the various metamodeling approaches onto the exposure problem.
- MC into Latin indices of each input parameter for the LARS PC-Kriging model.

**Assessment and comparison of model qualities**

Leave-one-out cross validation

**Conclusions et prospect**

- Use of LARS polynomials of chaos in the universal kriging model: Optimal metamodeling solution (at least as good as the best other solutions).
- In this case, significant improvement compared to the ordinary kriging model and similar performance with the classic sparse polynomial chaos.
- Use of the existing sequential DOE strategies oriented to the assessment of quantities with this optimal kriging model.

**References**

- Mathéron, G. "Le krigage universel". Cahiers du centre de morphologie mathématique, (1968)
- Blatman, G. and B. Sudret. "Adaptive sparse polynomial chaos expansion based on least angle regression." Journal of Computational Physics 230, no. 4 (2011): 2345-2367.



# As Final Conclusion

Dans la confusion trouver la simplicité  
De la discorde faire jaillir l'harmonie  
Au milieu de la difficulté se trouve  
l'opportunité

Albert Einstein,  
*Trois règles de travail*