SimTech Cluster of Excellence — University of Stuttgart, Germany

Uncertainty Quantification in Crack Propagation Simulations

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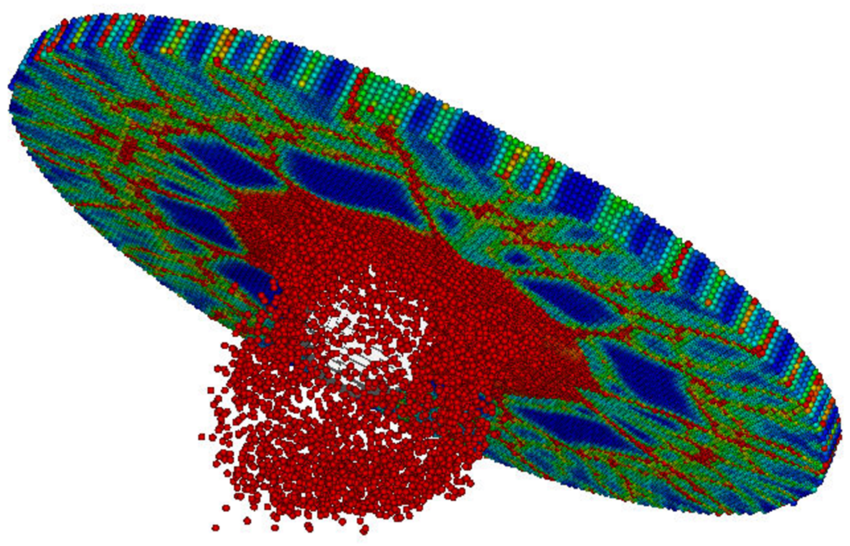
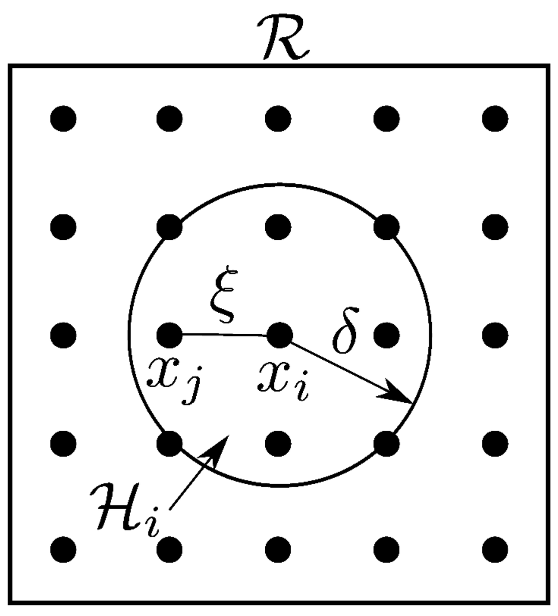
Motivation

- **Crack propagation** with Peridynamics in high-velocity impact simulations
- **Properties** of the UQ setting:
  1. non-intrusive
  2. expensive samples
  3. steep transition
  4. scale to large number of parameters

Crack Propagation and Peridynamics





- Particle-based simulation of cracks
- Equation of motion

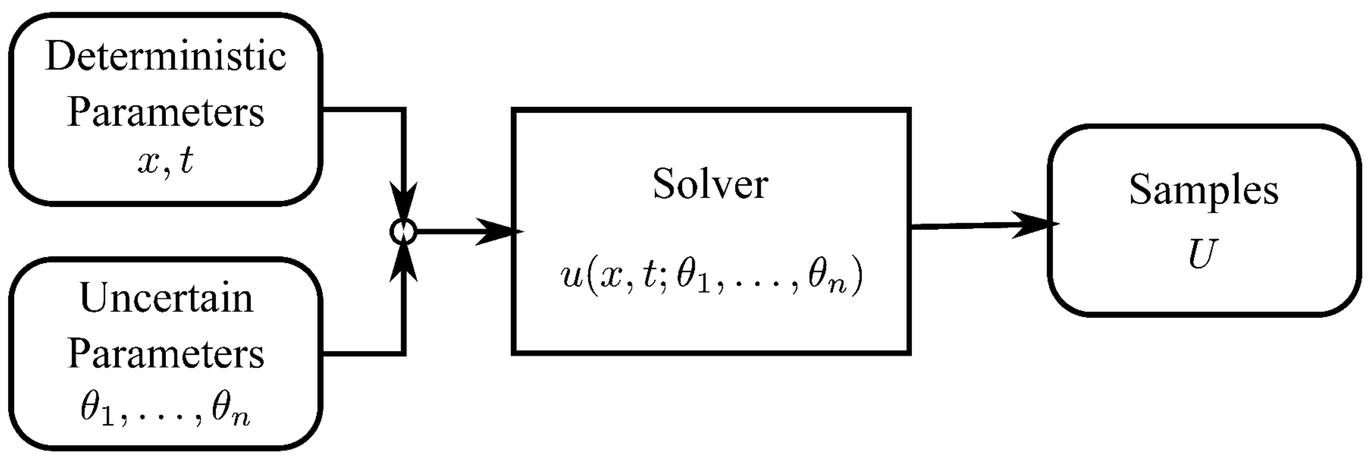
$$\rho(x_i)\ddot{u}(x_i,t) = \int_{\mathcal{H}_i} f(\xi,\eta)\,dV_{x_j} + b(x_i,t)\ . \tag{1}$$

- f is a **pair-wise force function** on pseudo-particles within **horizon** $\mathcal{H}_i := \{x_j : \|x_i - x_j\| \leq \delta\}$ for $\delta := 3\Delta x$ [3]
- Particles are connected through **elastic bonds**
- A bond breaks when its stretch exceeds a certain threshold
- Bond breaks represent **local damage** and form **cracks**
- **Problem:** properties of model parameters are unknown
- **Task:** find sensitivity measures for parameters by describing the lack of knowledge by probability density functions, and propagate it through the peridynamic model

Table: Range and distribution of the most interesting peridynamic model parameters.

Param.	Min	Max	Unit	Dist.	Description
Δx	0.4	0.6	mm	$\mathcal{U}(0.4, 0.6)$	particle density
α	0	1	-	$\mathcal{U}(0,1)$	elasticity
K	10^{12}	10^{20}	N/m^2	$\mathcal{U}(12, 20)$	projectile's magnitude of force

Non-intrusive Forward Propagation of Uncertainty

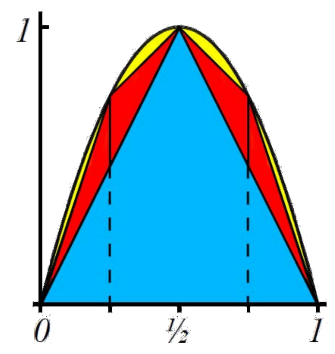


- Number of uncertain parameters is problem's dimensionality
- **Highly-dimensional** parameter space leads to **curse of dimensionality**
- **non-smooth** functional dependencies
- Main obstacles for conventional **non-stochastic** methods

Adaptive Sparse Grid Collocation [2]

- **Advantages:**
  1. cope with the curse of dimensionality to a large extent
  2. **non-intrusive** so easily applicable to many problems
  3. efficient and flexible analysis due to **explicit** representation
  4. resolve discontinuities: **adaptive refinement** and **local basis**
- Weighted sum of basis functions

$$f_{I_N}(\vec{\theta}) = \sum_{i \in I_N} v_i \phi_i(\vec{\theta}) \tag{2}$$



- **Hierarchical basis** and **tensor product**
- Number of grid points $\mathcal{O}(N \log_2(N)^{d-1})$ [1]
- L_2 convergence $\mathcal{O}(N^{-2}\log_2(N)^{3(d-1)})$ [1]

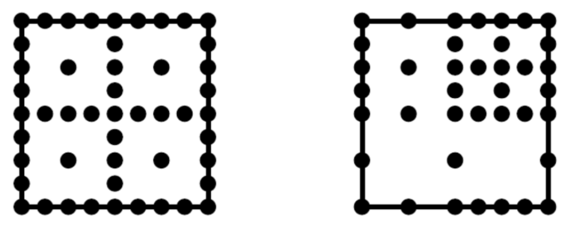


Figure: Sparse grid (left), adaptively refined sparse grid (right).

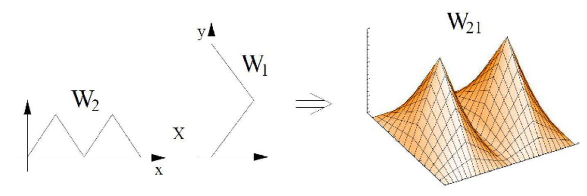


Figure: Tensor product basis example for two dimensions.

- Sobol indices

$$\mathbb{V}(u) = \sum_{i=1}^{d} \mathbb{V}_i(\theta_i) + \sum_{i=1}^{d}\sum_{i<j} \mathbb{V}_{i,j}(\theta_i,\theta_j) + \dots + \mathbb{V}_{1,\dots,d}(\theta_1,\dots,\theta_d) \tag{3}$$

- Adaptive refinement: minimize number of samples
  - **Idea:** hierarchical coefficients $\sim$ local change of the function
    1. absolute surplus: $\arg\max_{i \in I_N} |v_i|$
    2. expectation value: $\arg\max_{i \in I_N} |\mathbb{E}(f_{I_N \setminus \{i\}}) - \mathbb{E}(f_{I_N})| \Rightarrow \arg\max_{i \in I_N} |v_i| 2^{-|l|_1}$
    3. variance: $\arg\max_{i \in I_N} |w_i|$ where $f^2_{I_N}(\vec{\theta}) = \sum_{i \in I_n} w_i \phi_i(\vec{\theta})$ [2]

Results

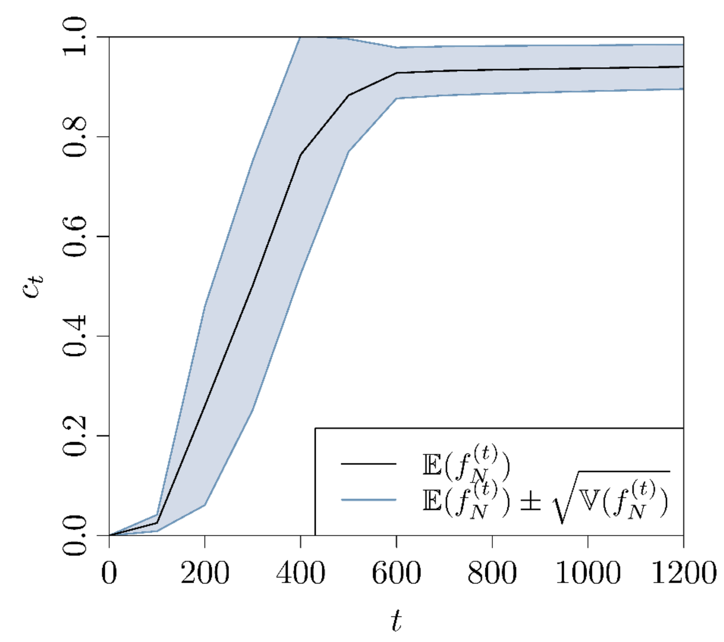


Figure: Expectation value and standard deviation of the total damage in the plate for $t \in \{0, 100, \dots, 1200\}$.

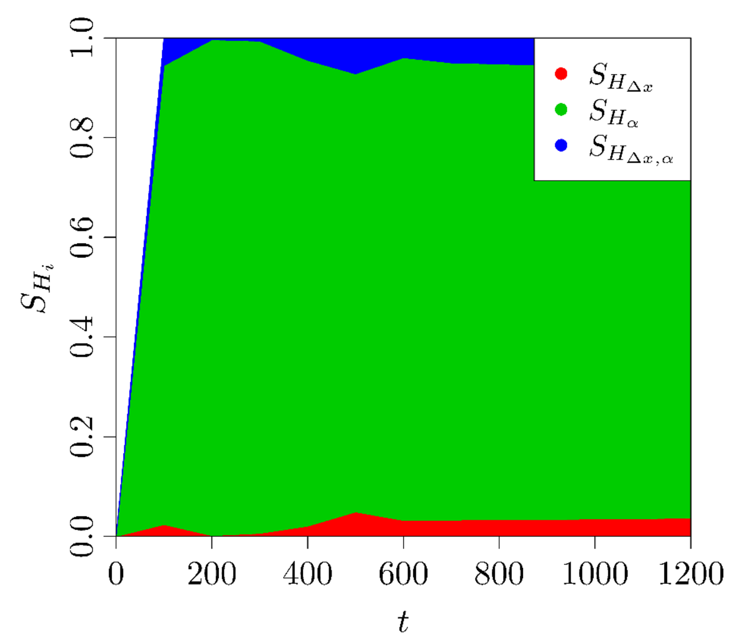


Figure: Sobol indices for Δx and α. Surprisingly large impact of α and rather low impact of Δx; large second-order interactions.

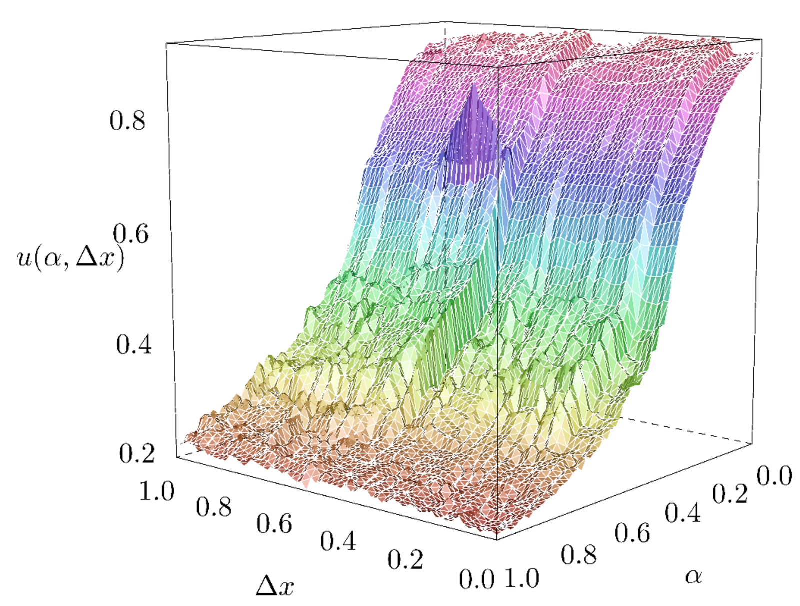


Figure: Sparse grid interpolant for damage at $t = 300$. It shows high oscillations in the lower and upper regime of α, discontinuities in direction of Δx and outliers.

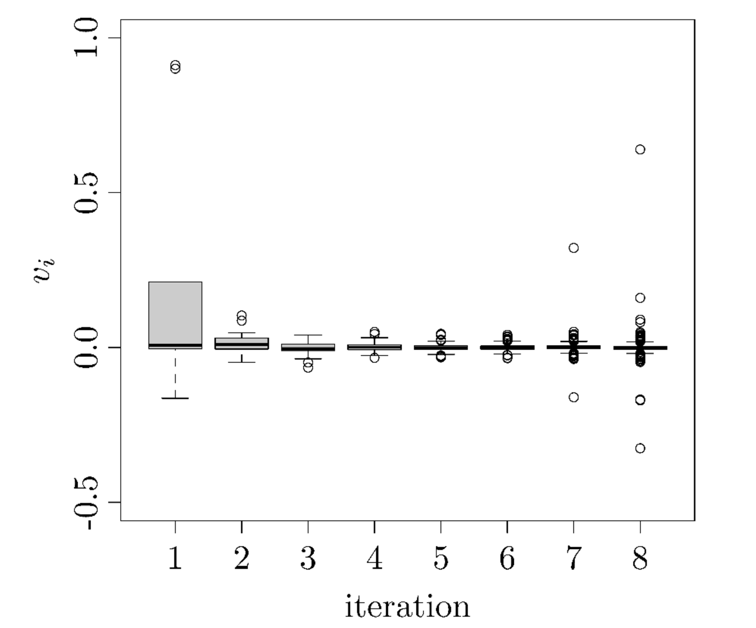


Figure: Box plot of the hierarchical coefficients of the regular sparse grid interpolant per iteration. Outliers and non-smooth regions have been detected.

Future Work

1. Add **bulk modulus** and the **critical stress intensity factor** to the simulation to study properties of non-existing materials
2. Compare Peridynamics with real physical experiments in the context of **speed of sound in materials**

[1] H.-J. Bungartz and M. Griebel. Sparse Grids. Acta Numerica, 13:1-123, 2004
[2] Ma, X. and Zabaras, N. An adaptive hierarchical sparse grid collocation algorithm for the solution of stochastic differential equations. JCP, (2009)
[3] S. Silling and E. Askari A meshfree method based on the peridynamic model of solid mechanics. Computers & Structures **83** (2005), no. 17-18, 1526–1535