Spatio-temporal hybrid Strauss hardcore point process and application

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Ph.D. expected duration: Oct. 2018 - Sep. 2021

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Abstract:

The theory of point processes is a branch of spatial statistics. A spatial (and spatio-temporal) point pattern, as a realization of a point process, is a collection of events for which locations (and times) of occurrence have been observed in a specified spatial region (and temporal period). Point patterns are often classified into three classes of single interaction structure: randomness, clustering, and inhibition that can be modeled for instance by Poisson process, Cox processes, and Gibbs processes, respectively.

Gibbs point processes offer a large class of models which allow any of the single structure and are studied by their probability density, defined with respect to a unit rate Poisson point process. In the literature, several spatio-temporal Gibbs point process models have been proposed such as the hardcore ([2]), Strauss ([3]), area-interaction ([4]), and Geyer ([5]) point processes.

Due to the capability of the Gibbs point processes to cover the prevalent structures, hybridization ([1]) can be an approach for introducing new Gibbs models which combine several structures at different scales. Given m densities $f_1, f_2, ..., f_m$ of Gibbs point processes, the hybrid density is defined as $f = cf_1 \times f_2 \times \cdots \times f_m$ where c is normalization constant ([1]). Such hybrid Gibbs models can then be applied to describe some complex phenomena for instance in epidemiology ([4]) and forestry ([5]).

In addition to these multi-structural patterns, hardcore distances also exist in many point patterns due to repulsion between points. In the spatial point process literature, the models are introduced mostly based on Gibbs models for modelling the point patterns with either clustering and inhibition or hardcore behavior on different scales simultaneously (e.g. [1]). Here we aim to investigate the spatio-temporal hybrid Strauss point process model including a hardcore component for modeling wildfire occurrences.

Let $\mathbf{x} = \{(\xi_1, t_1), , ..., (\xi_n, t_n)\}$ is a spatio-temporal point pattern where $(\xi_i, t_i) \in W = S \times T \subset \mathbb{R}^2 \times \mathbb{R}$. The cylindrical neighbourhood $C_r^q(u, v)$ centred at $(u, v) \in W = S \times T$ is defined as

$$C_r^q(u, v) = \{(a, b) \in S \times T : ||u - a|| \leq r, |v - b| \leq q\}, \tag{1}$$

where $r, q > 0$ are spatial and temporal radii. The Papangelou conditional intensity (a closely related concept to density functions) of a spatio-temporal point process on $W = S \times T$ with density f for $(u, v) \in W$ is defined by

$$\lambda((u, v)|\mathbf{x}) = \frac{f(\mathbf{x} \bigcup (u, v))}{f(\mathbf{x} \setminus (u, v))}, \tag{2}$$

with $a/0 := 0$ for all $a \geq 0$ ([2]).

The homogeneous spatio-temporal Strauss hardcore point process is defined by density

$$f(\mathbf{x}) = c\lambda^{n(\mathbf{x})} \gamma^{S_r^q(\mathbf{x})} \mathbb{1}\{||\xi - \xi'|| > h_s \text{ or } |t - t'| > h_t; \forall (\xi, t) \neq (\xi', t') \in \mathbf{x}\}, \tag{3}$$

where $c > 0$ is a normalizing constant, $\lambda > 0$ is activity parameter, γ is interaction parameter, $S_r^q(\mathbf{x}) = \sum_{(\xi,t)\neq(\xi',t')\in\mathbf{x}} \mathbb{1}\{||\xi - \xi'|| \leq r, |t - t'| \leq q\}$, r, q and $h_s, h_t > 0$ are spatial and temporal radii and spatial and temporal hardcore distances, respectively, $0 < h_s < r$ and $0 < h_t < q$ and $n(\mathbf{x})$ is the number of points in $\mathbf{x}$. The interaction parameter $0 < \gamma < 1$ reflects inhibition, while $\gamma > 1$ reflects clustering between points. When $\gamma = 1$, the density (3) corresponds to the density of a Poisson process.

By using hybridization approach ([1, 4, 5]), we define a multi-scale version of (3) with density

$$f(\mathbf{x}) = c \prod_{(\xi,t)\in\mathbf{x}} \lambda(\xi, t) \prod_{j=1}^{m} \gamma_j^{S_{r_j}^{q_j}(\mathbf{x})} \mathbb{1}\{||\xi' - \xi''|| > h_s \text{ or } |t' - t''| > h_t; \forall(\xi', t') \neq (\xi'', t'') \in \mathbf{x}\}, \quad (4)$$

where $0 < h_s < r_1 < \cdots < r_m$, $0 < h_t < q_1 < \cdots < q_m$. The function λ describes some spatio-temporal trend in point pattern that can be estimated using covariates. The Papangelou conditional intensity of model (4) for $(u, v) \notin \mathbf{x}$ is obtained

$$\lambda((u, v)|\mathbf{x}) = \lambda(u, v) \prod_{j=1}^{m} \gamma_j^{n[C_{r_j}^{q_j}(u,v);\mathbf{x}]} \prod_{(\xi,t)\in\mathbf{x}} \mathbb{1}\{(\xi, t) \notin C_{h_s}^{h_t}(u, v)\}, \quad (5)$$

where $n[C_r^q(\xi, t); \mathbf{x}] = \sum_{(u,v)\in\mathbf{x}} \mathbb{1}\{||u - \xi|| \leq r, |v - t| \leq q\}$ is the number of points in $\mathbf{x}$ which lying in $C_r^q(\xi, t)$. The function (5) can be used for inference and simulation of our model.

Gibbs point process models involve two types of parameters: *regular* and *irregular* parameters. A parameter is called *regular* if the log likelihood of density is a linear function of that parameter, *irregular* otherwise. In the Strauss hardcore point process (3), λ and γ are regular and r, q and h_s, h_t are irregular parameters. Irregular parameters can be predetermined by the user. Regular parameters can be estimated using the pseudo-likelihood or logistic likelihood method ([5]). Due to the advantage of the logistic likelihood over pseudo-likelihood for spatio-temporal Gibbs point processes ([4, 5]), we implement it for parameter estimation of our model.

As in [5], we implement a birth-death Metropolis-Hasting algorithm to simulate our model.

Finally, we illustrate the performance of our model both on simulations and real data application.

References

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Short biography – Morteza Raeisi currently works at the Laboratoire de Mathématiques d'Avignon (LMA), University of Avignon and BioSP research unit of INRAE (Institut national de recherche pour l'agriculture, l'alimentation et l'environnement) as a third year Ph.D. researcher. Morteza does research in point process models for complex spatio-temporal data: application to forest fires.