

# Nested sampling designs with small covering radii

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**Abstract:** The covering radius of the sampling design is a key factor in  $L_q$  error bounds for function approximation ( $1 \leq q \leq \infty$ ); see for instance [2, Prop. 3.2], [9, Chap.11]. Constructions of designs with small covering radii have received a lot of attention, in particular those forming regular patterns such as lattices [1]. Incremental constructions, although of major practical interest, have received less attention. There exist bounds on the covering radius (also called dispersion) for low discrepancy sequences used in Quasi-Monte Carlo methods [3, Chap.6], but they are extremely pessimistic and the performances of these constructions are rather deceiving.

Several incremental constructions will be presented in the talk, based on minimization of a relaxed version of the covering radius [6], on minimization of a Maximum-Mean-Discrepancy by kernel herding [7, 5], on greedy maximization of an integrated covering measure that defines a submodular set function [4], or on geometrical considerations leading to the greedy-packing algorithm and its boundary-phobic variants. In the later case, performance guarantees can be provided [8].

This work is partly supported by the ANR project INDEX (INcremental Design of EXperiments), nb. ANR-18-CE91-0007.

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