Modélisation Lagrangienne Stochastique pour des écoulements géophysiques

Mireille Bossy
Équipe Tosca, INRIA Sophia Antipolis

Atelier EDS et Incertitudes du GRD MASCOT-NUM
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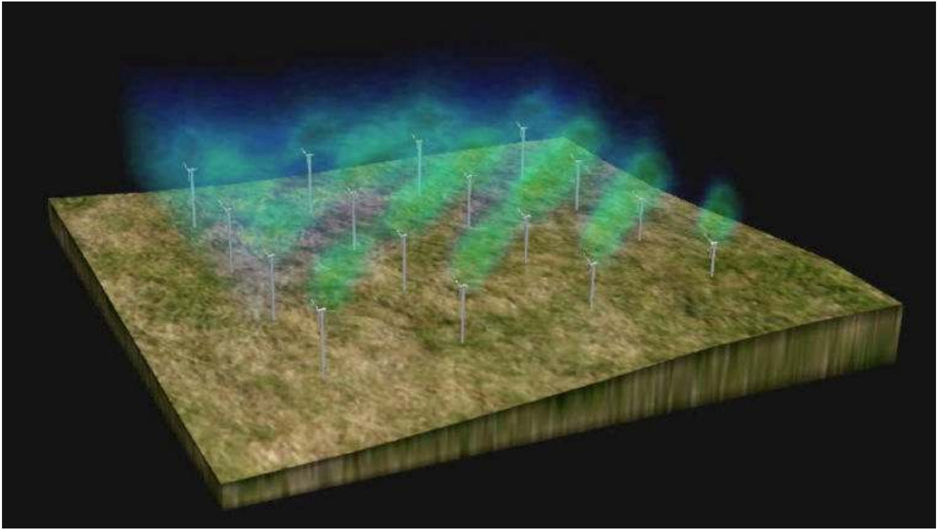
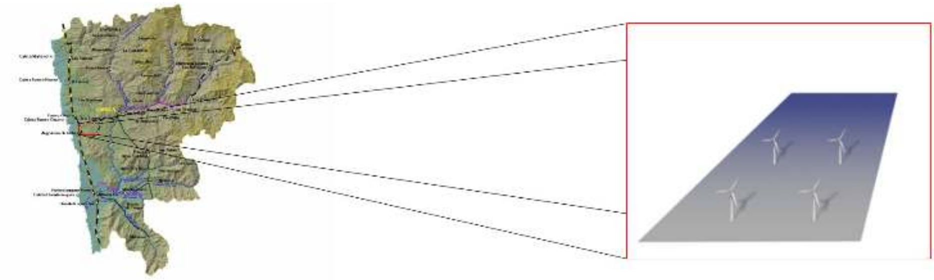
Compute the wind ...



... at a finer scale

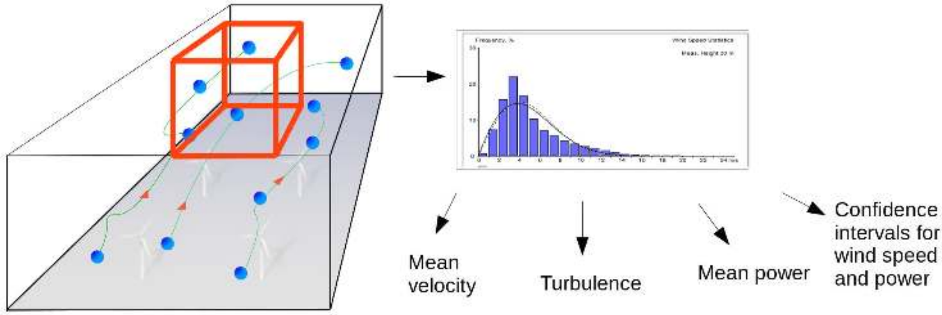


Mesoscale simulation delivers wind data on a coarse mesh

Downscaling

Windpos works on a fine mesh, using mesoscale data as boundary conditions

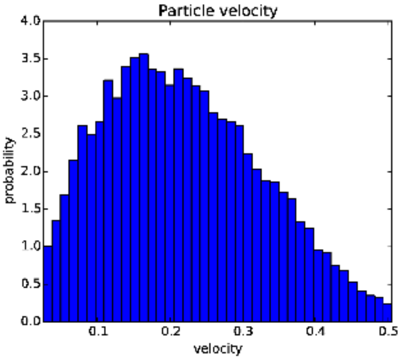
... with a stochastic Lagrangian approach



Lagrangian modeling
Fluid particle description as a stochastic process (X_t, U_t)

Lagrangian approach

The Law$(X_t, U_t) = \rho(t, x, u)dxdu$
is a local probability density of physical quantities (within the meaning of the statistical approach to turbulence).



Computation of local moment fields (with the ensemble averaged notation) :

$$\langle \mathscr{U} \rangle(t,x) = \frac{\int_{\mathcal{D}\times\mathbb{R}^3} u\, \rho(t,x,u)du}{\int_{\mathcal{D}\times\mathbb{R}^3} \rho(t,x,u)du} = \mathbb{E}\left[U_t | X_t = x\right]$$

$$\langle \mathscr{U}\, \mathscr{U}^t \rangle(t,x) = \frac{\int_{\mathcal{D}\times\mathbb{R}^3} u\, u^t; \rho(t,x,u)du}{\int_{\mathcal{D}\times\mathbb{R}^3} \rho(t,x,u)du} = \mathbb{E}\left[U_t\, U_t^t | X_t = x\right]$$

Lagrangian modeling for turbulent flow

On a given probability space $(\Omega, \mathcal{F}, \mathbb{P})$, consider the fluid particle state vector $(\mathrm{X}_t, \mathrm{U}_t, \psi_t)$ satisfying

$$
\begin{aligned}
d\mathrm{X}_t =& \mathrm{U}_t dt, \\
d\mathrm{U}_t =& \left[-\frac{1}{\varrho} \nabla_x \langle \mathscr{P} \rangle (t, \mathrm{X}_t) - G(t, X_t) \left(\mathrm{U}_t - \langle \mathscr{U} \rangle (t, \mathrm{X}_t) \right) \right] dt \\
& + \sqrt{C(t, \mathrm{X}_t) \varepsilon(t, \mathrm{X}_t)} dW_t, \\
d\psi_t =& D_1(t, \mathrm{X}_t, \psi_t) dt + D_2(t, \mathrm{X}_t, \psi_t) d\widetilde{W}_t.
\end{aligned}
$$

$\left(W, \widetilde{W}\right)$ is a $4D$-Brownian motion.

- $\langle \mathscr{U}^{(i)} \rangle (t, x)$, $\langle \mathscr{U}^{(i)} \mathscr{U}^{(j)} \rangle (t, x)$ are computed as conditional expectations.
- ε, C, G, D_1, D_2 are determined by a chosen Reynolds-averaged Navier-Stokes closure.

Statistical approach of turbulent flows

- The Reynolds averages (or ensemble averages) are expectations :

$$\langle \mathscr{U} \rangle(t,x) := \int_\Omega \mathscr{U}(t,x,\omega) d\mathbb{P}(\omega).$$

Reynolds decomposition

$$\mathscr{U}(t,x,\omega) = \langle \mathscr{U} \rangle(t,x) + \mathbf{u}(t,x,\omega),$$
$$\mathscr{P}(t,x,\omega) = \langle \mathscr{P} \rangle(t,x) + \mathbf{p}(t,x,\omega)$$

The random field $\mathbf{u}(t,x,\omega)$ is the turbulent part of the velocity.

- The incompressible Navier Stokes equation in $\mathbb{R}^3$, for the velocity field $(\mathscr{U}^{(1)}, \mathscr{U}^{(2)}, \mathscr{U}^{(3)})$ and the pressure $\mathscr{P}$, with constant mass density ϱ

$$\partial_t \mathscr{U} + (\mathscr{U} \cdot \nabla)\mathscr{U} = \nu \triangle \mathscr{U} - \frac{1}{\varrho} \nabla \mathscr{P}, \quad t > 0, \ x \in \mathbb{R}^3,$$
$$\nabla \cdot \mathscr{U} = 0, \quad t \geq 0, \ x \in \mathbb{R}^3,$$
$$\mathscr{U}(0,x) = \mathscr{U}_0(x), \ x \in \mathbb{R}^3.$$

The Reynolds-Averaged Navier-Stokes (RANS) Equation

Assuming Reynolds decomposition, with constant mass density ϱ, we get

$$\partial_t \langle \mathscr{U}^{(i)} \rangle + \sum_{j=1}^{3} \langle \mathscr{U}^{(j)} \rangle \partial_{x_j} \langle \mathscr{U}^{(i)} \rangle + \sum_{j=1}^{3} \partial_{x_j} \langle \mathbf{u}^{(i)} \mathbf{u}^{(j)} \rangle = \nu \triangle \langle \mathscr{U}^{(i)} \rangle - \frac{1}{\varrho} \partial_{x_i} \langle \mathscr{P} \rangle,$$

$$\nabla \cdot \langle \mathscr{U} \rangle = 0,\ t \geq 0,\ x \in \mathbb{R}^3,$$

$$\langle \mathscr{U} \rangle (0, x) = \langle \mathscr{U}_0 \rangle (x),\ x \in \mathbb{R}^3,$$

closed by a chosen parameterization or a model equation of the Reynolds stress tensor $(\langle \mathbf{u}^{(i)} \mathbf{u}^{(j)} \rangle = \langle \mathscr{U}^{(i)} \mathscr{U}^{(j)} \rangle - \langle \mathscr{U}^{(i)} \rangle \langle \mathscr{U}^{(j)} \rangle, i, j)$.

kinetic turbulent energy $k(t,x) := \sum_{i=1}^{3} \frac{1}{2} \langle \mathbf{u}^{(i)} \mathbf{u}^{(i)} \rangle (t,x)$

pseudo-dissipation $\varepsilon(t,x) := \nu \sum_{i=1}^{3} \sum_{k=1}^{3} \langle \partial_{x_k} \mathbf{u}^{(i)} \partial_{x_k} \mathbf{u}^{(i)} \rangle (t,x).$

Equation for the Reynolds stress $(\langle \mathbf{u}^{(i)}\mathbf{u}^{(j)}\rangle, i, j)$

$$\partial_t \langle \mathbf{u}^{(i)}\mathbf{u}^{(j)}\rangle + \left(\langle \mathscr{U}\rangle \cdot \triangle_x \langle \mathbf{u}^{(i)}\mathbf{u}^{(j)}\rangle\right) + \sum_{k=1}^{3} \partial_{x_k} \langle \mathbf{u}^{(i)}\mathbf{u}^{(j)}\mathbf{u}^{(k)}\rangle$$

$$= \underbrace{-\frac{1}{\varrho}\langle \mathbf{u}^{(j)}\partial_{x_i}\mathbf{p} + \mathbf{u}^{(i)}\partial_{x_j}\mathbf{p}\rangle}_{\text{velocity pressure gradient tensor } \Pi_{ij}} + \nu \nabla_x \langle \mathbf{u}^{(i)}\mathbf{u}^{(j)}\rangle$$

$$+ \underbrace{\nu \sum_{k=1}^{3} \langle \partial_{x_k}\mathbf{u}^{(i)}\partial_{x_k}\mathbf{u}^{(j)}\rangle}_{\text{dissipation tensor } \varepsilon_{ij}}$$

$$\underbrace{-\sum_{k=1}^{3}\left(\langle \mathbf{u}^{(i)}\mathbf{u}^{(k)}\rangle \partial_{x_k}\langle \mathscr{U}^{(i)}\rangle + \langle \mathbf{u}^{(j)}\mathbf{u}^{(k)}\rangle \partial_{x_k}\langle \mathscr{U}^{(j)}\rangle\right)}_{\text{turbulence production tensor } \mathcal{P}_{ij}}$$

Alternative viewpoint to compute the Reynolds stress

Let $\rho_E(t,x;V)$ be the probability density function of the (Eulerian) random field $\mathscr{U}(t,x)$, then

$$\langle \mathscr{U}^{(i)} \rangle(t,x) = \int_{\mathbb{R}^3} V^{(i)} \rho_E(t,x;V) dV,$$
$$\langle \mathscr{U}^{(i)} \mathscr{U}^{(j)} \rangle(t,x) = \int_{\mathbb{R}^3} V^{(i)} V^{(j)} \rho_E(t,x;V) dV.$$

The closure problem is reported on the PDE satisfied by the probability density function ρ_E.

In his seminal work, Stephen B. Pope proposes to model the PDF ρ_E with a Lagrangian description of the flow,

or equivalently with the Lagrangian probability density function.
$\triangleright$ This is the so-called PDF method.

Fluid particle model family

On a given probability space $(\Omega, \mathcal{F}, \mathbb{P})$, consider the fluid particle state vector $(\mathrm{X}_t, \mathrm{U}_t, \psi_t)$ satisfying

$$
\begin{aligned}
d\mathrm{X}_t =& \mathrm{U}_t dt \\
d\mathrm{U}_t =& \left[-\frac{1}{\varrho} \nabla_x \langle \mathscr{P} \rangle (t, \mathrm{X}_t) - G(t, X_t) \left(\mathrm{U}_t - \langle \mathscr{U} \rangle (t, \mathrm{X}_t) \right) \right] dt \\
&+ \sqrt{C(t, \mathrm{X}_t) \varepsilon(t, \mathrm{X}_t)} dW_t, \\
d\psi_t =& D_1(t, \mathrm{X}_t, \psi_t) dt + D_2(t, \mathrm{X}_t, \psi_t) d\widetilde{W}_t.
\end{aligned}
$$

$\left(W, \widetilde{W}\right)$ is a $4D$-Brownian motion.

One needs to

- compute the Eulerian fields $\langle \mathscr{U}^{(i)} \rangle (t, x)$, $\langle \mathscr{U}^{(i)} \mathscr{U}^{(j)} \rangle (t, x)$ and their derivatives
- compute ε, C, G, D_1, D_2 imposed by the chosen the RANS closure
- compute $\nabla_x \langle \mathscr{P} \rangle$
- introduce boundary condition

Compute the Reynolds averages $\langle \mathscr{U}^{(i)} \rangle$ and $\langle \mathscr{U}^{(i)} \mathscr{U}^{(j)} \rangle$

Let $\rho_L(t; x, V, \psi)$ be the probability density function of $(\mathrm{X}_t, \mathrm{U}_t, \psi_t)$.

Eulerian PDF versus Lagrangian PDF :
(Case of incompressible flow with constant mass density)

$$\rho_E(t, x; V, \phi) = \frac{\rho_L(t; x, V, \phi)}{\int_{\mathbb{R}^4} \rho_L(t; x, V, \psi) dV d\psi}$$

For any bounded measurable function $F(v)$,

$$\langle F(\mathscr{U}) \rangle (t, x) = \mathbb{E}\left(F(\mathrm{U}_t) / \mathrm{X}_t = x \right).$$

In particular,

$$\langle \mathscr{U}^{(i)} \rangle (t, x) = \int_{\mathbb{R}^4} V^{(i)} \frac{\rho_L(t; x, V, \phi)}{\int_{\mathbb{R}^4} \rho_L(t; x, U, \psi) dU d\psi} dV d\phi = \mathbb{E}\left(\mathrm{U}_t^{(i)} / \mathrm{X}_t = x \right).$$

Associated Fokker Planck Equation

(PDF description of fluid)

$\mathbb{P}\left((\mathrm{X}_t, \mathrm{U}_t, \psi_t) \in dxdvd\phi\right) := \rho_L(t; x, v, \phi)dxdvd\phi.$

$$\begin{aligned}\partial_t\rho_L + (v\cdot\nabla_x\rho_L) = \frac{1}{\varrho}\left(\nabla_x\langle\mathscr{P}\rangle(t,x)\cdot\nabla_v\rho_L\right) \\ - \nabla_v\cdot\left(G(t,x)(\langle\mathscr{U}\rangle(t,x) - v)\rho_L\right) + \frac{1}{2}C(t,x)\varepsilon(t,x)\triangle_v\rho_L \\ - \partial_\phi(D^1(t,x,\phi)\rho_L) + \frac{1}{2}\partial^2_{\phi\phi}\left(D^2(t,x,\phi)\rho_L\right)\end{aligned}$$

- Integrating w.r.t. $dvd\phi$ gives the equation for the conservation of mass, $\varrho(t,x) = \int\rho_L(t; x, v, \phi)dvd\phi$

$$\partial_t\int\rho_L dvd\phi + \nabla_x\cdot\left(\frac{\int v\rho_L dvd\phi}{\int\rho_L dvd\phi}\int\rho_L dvd\phi\right) = 0 \Longleftrightarrow \partial_t\varrho + \nabla_x\cdot(\varrho\langle\mathscr{U}\rangle) = 0.$$

- Integrating w.r.t. $vdvd\phi$ gives the RANS equation.
- Integrating w.r.t. $vv^t dvd\phi$ gives the Reynolds stress closure, according to C, G, D^1, D^2

Outline

- ► Wellposedness
- ▷ Algorithms and numerical analysis
- ▷ Application : computation of wind circulation around mills

The SDE versus PDE problem

For any arbitrary finite $T > 0$, we are interested in $((X_t, U_t);\ 0 \leq t \leq T)$, solving

$$\begin{cases} X_t = X_0 + \int_0^t U_s\, ds, \\ U_t = U_0 + \int_0^t B[X_s; \rho_s] ds + \sigma W_t, \\ \rho(t,x,v) \text{ is the probability density} \\ \qquad \text{of } (X_t, U_t) \text{ for all } t \in (0,T], \end{cases} \qquad B[x;\rho] = \begin{cases} \dfrac{\int_{\mathbb{R}^d} b(v)\rho(t,x,v)dv}{\int_{\mathbb{R}^d} \rho(t,x,v)dv}, \text{ if } \int_{\mathbb{R}^d} \rho(t,x,v)dv \neq 0 \\ 0 \text{ otherwise,} \end{cases}$$

$(W_t, t \geq 0)$ is a standard $\mathbb{R}^d$-Brownian motion ; the diffusion $\sigma > 0$ is a constant ;

$$\begin{cases} \partial_t \rho + (u \cdot \nabla_x \rho) + (B[\cdot\ ; \rho] \cdot \nabla_u \rho) - \dfrac{\sigma^2}{2} \triangle_u \rho = 0 \ \text{ in } (0,T) \times \mathcal{D} \times \mathbb{R}^d, \\ \rho(0,x,u) = \rho_0(x,u) \ \text{ on } \mathcal{D} \times \mathbb{R}^d, \end{cases}$$

✓ Mean field limit of an interacting particles system (propagation of chaos) ;
✓ Conditional McKean-Vlasov Fokker-Planck equation.

- ▷ Add some boundary conditions.
- ▷ Add a mass constraint.

Well-posedness of a toy model

$$\begin{cases} \mathrm{X}_t = \mathrm{X}_0 + \int_0^t \mathrm{U}_s ds, \\ \mathrm{U}_t = \mathrm{U}_0 + \int_0^t B\left[\mathrm{X}_s, \mathrm{U}_s; \rho_s\right] ds + \int_0^t \sigma(s, \mathrm{X}_s, \mathrm{U}_s)\, dW_s, \\ \mathbb{P}((\mathrm{X}_t, \mathrm{U}_t) \in dxdu) = \rho_t(x,u)dxdu,\ t \in [0,T], \end{cases}$$

If $b : \mathbb{R}^{2d} \to \mathbb{R}^d$ is bounded, by the Girsanov theorem, any weak solution $(\mathrm{X}_t, \mathrm{U}_t, t \in [0,T])$ has a strictly positive density $(\rho_t(x,u), t \in [0,T])$,

and $\quad \mathbb{E}\left[b(u, \mathrm{U}_t) \middle| \mathrm{X}_t = x\right] = B[x, u; \rho_t].$

when $\quad B[x,u;\gamma] = \begin{cases} \dfrac{\int_{\mathbb{R}^d} b(u,v)\gamma(x,v)dv}{\int_{\mathbb{R}^d} \gamma(x,v)dv}, & \text{if } \displaystyle\int_{\mathbb{R}^d} \gamma(x,v)dv \neq 0, \\ 0, & \text{elsewhere} \end{cases}$

Well-posedness of a toy model

Theorem [Bossy, Jabir, Talay 2011

The simplified Langevin model has a unique weak solution, under the following hypotheses :

- σ is bounded and strongly elliptic : for $a := \sigma\sigma^*$, there exists $\lambda > 0$ s.t. for all $t \in (0,T],\ x,\ u,\ v \in \mathbb{R}^d$,

$$\frac{|v|^2}{\lambda} \leq \sum_{i,j=1}^{d} a^{(i,j)}(t,x,u)v_i v_j \leq \lambda |v|^2 .$$

- For all $1 \leq i,j \leq d$, $a^{(i,j)}$ is Hölder continuous in the following sense : $\alpha \in (0,1]$

$$\begin{aligned} &|a^{(i,j)}(t,x,u) - a^{(i,j)}(s,y,v)| \\ &\leq C(|t-s|^{\frac{\alpha}{2}} + |x-y-v(t-s)|^{\alpha} + |u-v|^{\alpha}). \end{aligned}$$

- $b \in C_b(\mathbb{R}^{2d}, \mathbb{R}^d)$, and $(\mathrm{X}_0, \mathrm{U}_0)$ s.t. $\mathbb{E}_{\mathbb{P}}\left[\|\mathrm{X}_0\|_{\mathbb{R}^d} + \|\mathrm{U}_0\|^2_{\mathbb{R}^d}\right] < +\infty$.

The smoothed system

$$\begin{cases} X_t^\varepsilon = X_0 + \int_0^t U_s^\varepsilon \, ds, \\ U_t^\varepsilon = U_0 + \int_0^t B_\varepsilon \left[X_s^\varepsilon, U_s^\varepsilon; \rho_s^\varepsilon\right] ds + \int_0^t \sigma(s, X_s^\varepsilon, U_s^\varepsilon)\, dW_s, \\ \text{where } \mathcal{L}aw(X_t^\varepsilon, U_t^\varepsilon) = \rho_t^\varepsilon(x,u) dx\, du \end{cases}$$

and where for all non–negative γ in $L^1(\mathbb{R}^{2d})$,

$$B_\varepsilon\left[x, u; \gamma\right] = \frac{\int_{\mathbb{R}^{2d}} b(v,u) K_\varepsilon(x-y)\gamma(y,v)\, dy\, dv}{\int_{\mathbb{R}^{2d}} K_\varepsilon(x-y)\gamma(y,v)\, dy\, dv + \varepsilon},$$

for a given regularization $K_\varepsilon \in C_c^1(\mathbb{R}^d)$ of the Dirac mass in $\mathbb{R}^d$.

$$\begin{cases} X_t^\varepsilon = X_0 + \int_0^t U_s^\varepsilon ds, \\ U_t^\varepsilon = U_0 + \displaystyle\int_0^t \frac{\mathbb{E}\left[b(v, U_s^\varepsilon) K_\varepsilon(x - X_s^\varepsilon)\right]\Big|_{x = X_s^\varepsilon, v = U_s^\varepsilon}}{\mathbb{E}\left[K_\varepsilon(x - X_s^\varepsilon)\right]\Big|_{x = X_s^\varepsilon} + \varepsilon} ds + \int_0^t \sigma(s, X_s^\varepsilon, U_s^\varepsilon)\, dW_s, \\ \qquad t \in [0, T]. \end{cases}$$

Interacting particles system

$B_\varepsilon[x,u;\rho_t^\varepsilon]$ is approximated by

$$B_\varepsilon\left[x,u;\mu_t^\varepsilon\right]=\frac{\displaystyle\frac{1}{N}\sum_{j=1}^{N}b(\mathrm{U}_t^{j,\varepsilon},u)K_\varepsilon(x-\mathrm{X}_t^{j,\varepsilon})}{\displaystyle\frac{1}{N}\sum_{j=1}^{N}K_\varepsilon(x-\mathrm{X}_t^{j,\varepsilon})+\varepsilon},$$

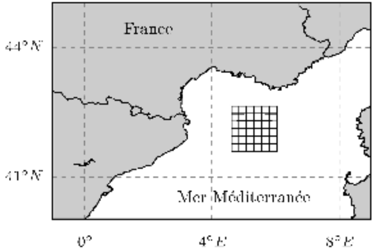
where $\mu^\varepsilon=\dfrac{1}{N}\sum_{j=1}^{N}\delta_{(\mathrm{X}_\cdot^{j,\varepsilon},\mathrm{U}_\cdot^{j,\varepsilon})}$ is the empirical measure of the following set of particles : given $(W_t^i,t\geq 0;i\geq 1)$, a family of d-dimensional Brownian motions,

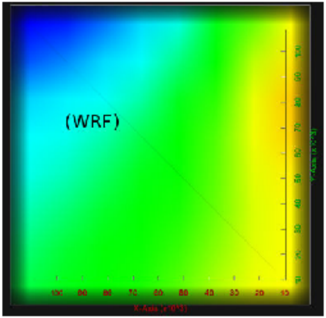
$$\begin{cases}\mathrm{X}_t^{i,\varepsilon}=\mathrm{X}_0^i+\int_0^t\mathrm{U}_s^{i,\varepsilon}ds,\\ \mathrm{U}_t^{i,\varepsilon}=\mathrm{U}_0^i+\displaystyle\int_0^t\frac{\frac{1}{N}\sum_{j=1}^{N}b(\mathrm{U}_s^{j,\varepsilon},\mathrm{U}_s^{i,\varepsilon})K_\varepsilon(\mathrm{X}_s^{j,\varepsilon}-\mathrm{X}_s^{j,\varepsilon})}{\frac{1}{N}\sum_{j=1}^{N}K_\varepsilon(\mathrm{X}_s^{i,\varepsilon}-\mathrm{X}_s^{j,\varepsilon})+\varepsilon}dt+\int_0^t\sigma(s,\mathrm{X}_s^{i,\varepsilon},\mathrm{U}_s^{i,\varepsilon})\,dW_s^i,\end{cases}$$

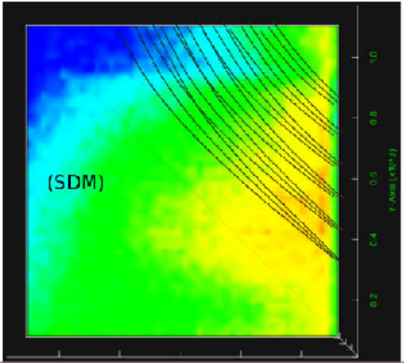
The law of $(\mathrm{X}^{i,\varepsilon},\mathrm{U}^{i,\varepsilon},i=1,\ldots,N)$ propagates Chaos when N tends to infinity.

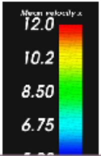
The downscaling problem

Collaboration with IPSL





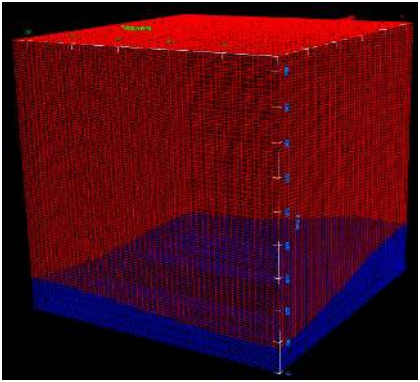




Comparison WRF and SDM on flat terrain ;
horizontal visualization of the u component of the wind at the altitude 400 m ;
date : T_0 plus 4 days .

Numerical framework

Our computational domain $\mathcal{D}$ corresponds to a given set of cell of a NWP solver.



Top and lateral (in red) :
mean-Dirichlet boundary condition

$$\forall x \in \partial\mathcal{D},\ \langle \mathscr{U} \rangle(t,x) = V_{\text{ext}}(t,x)$$

where V_{ext} is the WRF forcing.

Bottom boundary condition (in blue) necessities to introduce a wall law model.

The boundary condition in the PDF approach

$\forall x \in \partial\mathcal{D}$,

$$\langle \mathscr{U} \rangle(t,x) = V_{\text{ext}}(t,x).$$

$$\frac{\int_{\mathbb{R}^d} v\rho_\ell(t,x,v)dv}{\int_{\mathbb{R}^d} \rho_\ell(t,x,v)dv} = V_{\text{ext}}(t,x).$$

$$\begin{aligned}
& \int_{\mathbb{R}^d} v\rho_\ell(t,x,v,)dv = \int_{\mathbb{R}^d} V_{\text{ext}}(t,x)\rho_\ell(t,x,v)dv \\
\Leftrightarrow \quad & \int_{\mathbb{R}^d} v\rho_\ell(t,x,v,)dv = \int_{\mathbb{R}^d} v\rho_\ell(t,x,v+2(V_{\text{ext}}(t,x)-v))dv \\
\Uparrow \quad &
\end{aligned}$$

$$\rho_\ell(t,x,v,) = \rho_\ell(t,x,v+2(V_{\text{ext}}(t,x)-v)),\ \forall v \in \mathbb{R}^d$$

leads to specular boundary condition with jump on $\partial\mathcal{D}$ for ρ_ℓ ...

The Stochastic Downscaling Method (SDM)

Let $\mathcal{D}$ be an open set of $\mathbb{R}^3$, and a velocity V_{ext} given at $\partial\mathcal{D}$:

$$\begin{cases} d\mathrm{X}_t = & \mathrm{U}_t dt, \\ d\mathrm{U}_t = & \Big[-\frac{1}{\varrho}\nabla\langle\mathscr{P}\rangle(t,\mathrm{X}_t) \\ & \quad -\left(\frac{1}{2}+\frac{3}{4}C_0\right)\frac{\varepsilon(t,\mathrm{X}_t)}{k(t,\mathrm{X}_t)}\left(\mathrm{U}_t-\langle\mathscr{U}\rangle(t,\mathrm{X}_t)\right)\Big]\,dt \\ & +\sqrt{C_0\varepsilon(t,\mathrm{X}_t)}dW_t \\ & +\sum_{0<s\leq t} 2\left(V_{\text{ext}}(s,\mathrm{X}_s)-\mathrm{U}_{s^-}\right)\mathbb{1}_{\{\mathrm{X}_s\in\partial\mathcal{D}\}}. \end{cases}$$

The jump term should ensure that

$$\langle\mathscr{U}\rangle(t,x) = V_{\text{ext}}(t,x), \forall x \in \partial\mathcal{D}.$$

Well-posedness of the SLM with the no-permeability boundary condition $(\langle\mathscr{U}\rangle(t,x)\cdot n_{\mathcal{D}}(x))=0$

$$\begin{cases} X_t = X_0 + \int_0^t U_s\,ds, \\ U_t = U_0 + \int_0^t B[X_s;\rho(s)]ds + \sigma W_t - \sum_{0<s\leq t} 2\left(U_{s^-}\cdot n_{\mathcal{D}}(X_s)\right) n_{\mathcal{D}}(X_s)\mathbb{1}_{\{X_s\in\partial\mathcal{D}\}}, \\ \rho(t) \text{ is the probability density of } (X_t,U_t) \text{ for all } t\in(0,T], \end{cases}$$

$Q_T = (0,T)\times\mathcal{D}\times\mathbb{R}^d$ and $\Sigma_T = (0,T)\times\partial\mathcal{D}\times\mathbb{R}^d$

Definition - Trace of the density along Σ_T

$\gamma(\rho):\Sigma_T\to\mathbb{R}$ is the trace of $(\rho(t);\ t\in[0,T])$ along Σ_T if it is nonnegative and satisfies, for all t in $(0,T]$, f in $C_c^\infty\left(\overline{Q_t}\right)$:

$$\begin{aligned}&\int_{\Sigma_t}(u\cdot n_{\mathcal{D}}(x))\,\gamma(\rho)(s,x,u)f(s,x,u)\,ds\,d\sigma_{\partial\mathcal{D}}(x)\,du\\ &= \int_{\mathcal{D}\times\mathbb{R}^d}(f(0)\rho(0)-f(t)\rho(t)) + \int_{Q_t}\left(\partial_s f + u\cdot\nabla_x f + B[\cdot;\rho.]\cdot\nabla_u f + \frac{\sigma^2}{2}\triangle_u f\right)\rho(s)\end{aligned}$$

and, for $dt\otimes d\sigma_{\partial\mathcal{D}}$ a.e. (t,x) in $(0,T)\times\partial\mathcal{D}$,

$$\int_{\mathbb{R}^d}|(u\cdot n_{\mathcal{D}}(x))|\gamma(\rho)(t,x,u)\,du < +\infty, \qquad \int_{\mathbb{R}^d}\gamma(\rho)(t,x,u)\,du > 0.$$

Theorem [Bossy and Jabir 14, 15]

Under (*H*), there exists a unique solution in law to the SLM with jumps in $\mathcal{M}(\mathcal{C}([0,T];\overline{\mathcal{D}}) \times \mathbb{D}([0,T];\mathbb{R}^d))$, and this law admits time marginal densities $\rho_t = \mathbb{P} \circ (x(t), u(t))^{-1}$ in $L^2(\omega; \mathcal{D} \times \mathbb{R}^d)$, for all $t \in [0,T]$, that solves in $V_1(\omega; Q_T)$

$$\begin{cases} \partial_t \rho(t,x,u) + u \cdot \nabla_x \rho(t,x,u) - \dfrac{\sigma^2}{2} \triangle_u \rho(t,x,u) = -\left(B[x;\rho(t)] \cdot \nabla_u \rho(t,x,u)\right) \text{ in } Q_T \\ \rho(0,x,u) = \rho_0(x,u) \text{ in } \mathcal{D} \times \mathbb{R}^d, \\ \gamma^-(\rho)(t,x,u) = \gamma^+(\rho)(t,x,u - 2(u \cdot n_{\mathcal{D}}(x)) n_{\mathcal{D}}(x)) \text{ in } \Sigma_T^+, \end{cases}$$

The trace $\gamma(\rho)$ (in the sense of the previous definition) satisfies the no-permeability boundary condition

$$\mathbb{E}\{(U_t \cdot n_{\mathcal{D}}(X_t)) | X_t = x\} = \frac{\int_{\mathbb{R}^d} (u \cdot n_{\mathcal{D}}(x)) \gamma(\rho)(t,x,u)\, du}{\int_{\mathbb{R}^d} \gamma(\rho)(t,x,u)\, du} = 0, \quad dt \otimes d\sigma_{\partial\mathcal{D}}$$

-a.e. on $(0,T) \times \partial\mathcal{D}$.

The associated smoothed N-particles system propagates chaos.

Hypotheses (H)

(H_{Langevin}) for the construction of the linear Langevin process
(H_{MVFP}) for the well-posedness of the Vlasov-Fokker-Planck equation

- (H_{Langevin})-(i) (X_0, U_0) is distributed according to the initial law μ_0 having its support in $\mathcal{D} \times \mathbb{R}^d$ and such that $\int_{\mathcal{D}\times\mathbb{R}^d} \left(|x|^2 + |u|^2\right) \mu_0(dx, du) < +\infty$.
- (H_{Langevin})-(ii) The boundary $\partial\mathcal{D}$ is a compact $\mathcal{C}^3$ submanifold of $\mathbb{R}^d$.

- (H_{MVFP})-(i) $b : \mathbb{R}^d \to \mathbb{R}^d$ is a bounded continuous function.
- (H_{MVFP})-(ii) The initial law μ_0 has a density ρ_0 in the weighted space $L^2(\omega, \mathcal{D} \times \mathbb{R}^d)$ with $\omega(u) := (1 + |u|^2)^{\frac{\alpha}{2}}$ for some $\alpha > d + 3$.
- (H_{MVFP})-(iii) There exist two measurable functions $\underline{P}_0, \overline{P}_0 : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ such that

$$0 < \underline{P}_0(|u|) \leq \rho_0(x, u) \leq \overline{P}_0(|u|), \text{ a.e. on } \mathcal{D} \times \mathbb{R}^d;$$

$$\text{and} \quad \int_{\mathbb{R}^d} (1 + |u|)\omega(u)\overline{P}_0^2(|u|)du < +\infty.$$

Incompressible stochastic Lagrangian model in the torus

Goal : well-posedness for the following process in $\mathbb{T}^d \times \mathbb{R}^d$:

$$X_t = [X_0 + \int_0^t U_s ds] \text{ modulo } \mathbf{1}$$
$$U_t = U_0 + \int_0^t \left[\beta \left(\langle U_s \rangle - \alpha U_s \right) - \frac{\nabla P(s, X_s)}{\rho(s, X_s)} \mathbb{1}_{\{\rho(s,X_s)>0\}} \right] ds + \sigma W_t.$$

with

$$\mathcal{L}aw(X_t) = \rho(t,x)dx, \quad \langle U_t \rangle = \mathbb{E}(U_t | X_t)$$

and

$$-\triangle_x P(t,x) = \sum_{i,j=1}^{d} \partial^2_{ij} \left(\mathbb{E}(U^i_t U^j_t | X_t = x) \rho(t,x) \right), \quad (t,x) \in \mathbb{R}_+ \times \mathbb{T}^d.$$

Incompressible stochastic Lagrangian model in the torus

Lemma [in Bossy, Fontbona, Jabin and Jabir 2012]

Assume that the previous nonlinear SDE has a solution (X, U) such that

$$\mathbb{E}|U_t|^2 < \infty \; \forall t \in [0,T], \quad \mathbb{E}\int_0^T |U_s|^2 ds < \infty \text{ and } \int_0^T \int_{\mathbb{T}^d} |\nabla P(s,x)| dx ds < \infty.$$

Assume also that for all $1-$periodic function $\varphi : \mathbb{R}^d \to \mathbb{R}$ of class C^1 we have $\mathbb{E}(\nabla\varphi(X_0) \cdot U_0) = 0$. Then,

a) $\mathbb{E}(\nabla\varphi(X_t) \cdot U_t) = 0$ for all $t \in [0,T]$,

b) the process $(X_t, t \in [0,T])$ is stationary.

Existence result for the Vlasov Fokker Planck equation ?

Local existence of analytical solutions

In Bossy Fontbona, Jabin and Jabir 2012

$$
\begin{cases}
\partial_t f(t,x,u) + u \cdot \nabla_x f(t,x,u) = \dfrac{\sigma^2}{2} \triangle_u f(t,x,u) + \beta\, d f(t,x,u) + \beta u \cdot \nabla_u f(t,x,u) \\
\qquad + \nabla_u f(t,x,u) \cdot \left(\nabla P(t,x) - \beta\alpha \displaystyle\int_{\mathbb{R}^d} v f(t,x,v)\, dv \right) = 0 \ \text{ on } (0,T] \times \mathbb{T}^d \times \mathbb{R}^d, \\
f(t=0) = f_0 \ \text{ on } \mathbb{T}^d \times \mathbb{R}^d, \\
\triangle P(t,x) = -\partial_{x_i x_j} \displaystyle\int_{\mathbb{R}^d} v_i v_j f(t,x,v) dv, \ \text{ on } [0,T] \times \mathbb{T}^d,
\end{cases}
$$

Theorem (d=1)

Let $\bar{\lambda} > 0$ and $s \geq d+2$ be an even integer. There exists $\kappa_0 = \kappa_0(\bar{\lambda}, s)$ and $r \mapsto \kappa_1(r, \bar{\lambda}, s) \geq 0$ s.t. if $f_0 : \mathbb{T}^d \times \mathbb{R}^d \to \mathbb{R}$ of class $\mathcal{C}^\infty$ and $T > 0$ satisfy :

- $\int_{\mathbb{R}^d} f_0(x,u) du = 1$ and $\nabla_x \cdot \int_{\mathbb{R}^d} u f_0(x,u) du = 0$ for all $x \in \mathbb{T}$,
- $\|(1+|u|^2)^{\frac{s}{2}} D_x^l D_u^k f_0\|_\infty \leq \frac{C_0 (|k|+m)!(|l|+n)!}{\bar{\lambda}^{|k|+|l|}}$ for some $n, m \in \mathbb{N}$, all pair of multiindexes $k, l \in \mathbb{N}^{\mathbb{N}}$ and a constant $C_0 < \kappa_0(\bar{\lambda}, s)$, and
- $T < \kappa_1(C_0, \bar{\lambda}, s)$,

then, a solution f of class $\mathcal{C}^{1,\infty}$ exists.

Outline

▷ Wellposedness

► Algorithms and numerical analysis

▷ Application : computation of wind circulation around mills

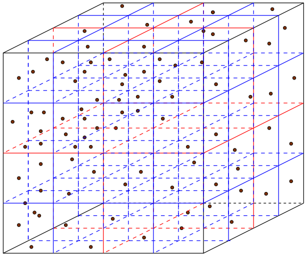
The computational space is divided in cells

We use a *Particles in cell* (PIC) technique to compute the Eulerian fields (mean fields) $\langle \mathscr{U}^{(i)} \rangle(t,x)$, $\langle \mathscr{U}^{(i)} \rangle \mathscr{U}^{(j)}(t,x)$ at the center of each cell :

We introduce N_p particles $(\mathrm{X}_t^{k,N_p}, \mathrm{U}_t^{k,N_p})$ in $\mathcal{D}$.

Each cell $\mathcal{C}$ contains N_{pc} particles by constant mass density constraint.

$K(.,x) = \mathbb{1}(.,\mathcal{C}(x))$.



$$\langle F(\mathscr{U}) \rangle(t,x) \simeq \frac{1}{N_p} \sum_{k=1}^{N_p} F\left(\mathrm{U}_t^{k,N_p}\right) K(\mathrm{X}_t^{k,N_p},x) \Big/ \frac{1}{N_p} \sum_{k=1}^{N_p} K(\mathrm{X}_t^{k,N_p},x)$$

$$\sum_{k=1}^{N_p} K(\mathrm{X}_t^{k,N_p},x) = N_{pc}$$

The interacting particles system

For $j = 1, \ldots, N_p$

$$\begin{cases} dX_t^{j,N_p} = U_t^{j,N_p} dt, \\ dU_t^{(i),j,N_p} = -\frac{1}{\varrho} \frac{\partial \langle \mathscr{P} \rangle}{\partial x_i}(t, X_t^{j,N_p}) dt \\ \qquad + D_U(t, X_t^{j,N_p}) dt + B_U(t, X_t^{j,N_p}) dW_t^{(i),j,N_p} \\ \qquad + \text{jump terms for boundary condition}, \quad i \in \{1, 2, 3\}. \end{cases}$$

- The coefficients D_U, B_U depend on the particles approximations of $\langle \mathscr{U} \rangle$, $\langle \mathscr{U}^{(i)} \mathscr{U}^{(j)} \rangle$, and their derivatives.

- $-\frac{1}{\varrho} \frac{\partial \langle \mathscr{P} \rangle}{\partial x_i}(t, X_t^{j,N_p})$ ensures that $\nabla \cdot \langle \mathscr{U} \rangle = 0$ and maintains the mass density constant.

$$\nabla^2 \langle \mathscr{P} \rangle = -\frac{\partial^2 \langle \mathscr{U}^{(i)} \mathscr{U}^{(j)} \rangle}{\partial x_i \partial x_j}$$

Algorithm

Fractional step method : $n\Delta t \longrightarrow (n+1)\Delta t$ (Pope 85 revisited)

Step 1 : For $n\Delta t \leq t \leq (n+1)\Delta t$, $(\mathrm{X}^{j,N_p}_{n\Delta t}, \mathrm{U}^{(i),j,N_p}_{n\Delta t}, j = 1, \ldots, N_p)$, given

$$\begin{cases} d\widetilde{\mathrm{X}}^{j,N_p}_t = \widetilde{\mathrm{U}}^{j,N_p}_t dt, \\ d\widetilde{\mathrm{U}}^{(i),j,N_p}_t = -\frac{1}{\varrho}\frac{\partial\langle\mathscr{P}\rangle}{\partial x_i}(t, \widetilde{\mathrm{X}}^{j,N_p}_t)dt \\ \qquad + D_{\widetilde{\mathrm{U}}}(t, X^{j,N_p}_t)dt + B_{\widetilde{\mathrm{U}}}(t, X^{j,N_p}_t)dW^{(i),j,N_p}_t \\ \qquad + \text{jump terms}, \quad i \in \{1,2,3\} \end{cases}$$

Step 2 : Correction of the particles positions $\widetilde{\mathrm{X}}^{j,N_p}_{(n+1)\Delta t} \longrightarrow \mathrm{X}^{j,N_p}_{(n+1)\Delta t}$ that maintains the (discrete) uniform distribution, by solving a (discrete) optimal transport problem.

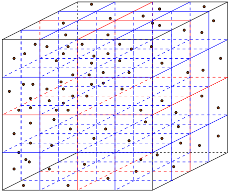
Step 3 : Correction of the particles velocity $\widetilde{\mathrm{U}}^{j,N_p}_{(n+1)\Delta t} \longrightarrow \mathrm{U}^{j,N_p}_{(n+1)\Delta t}$ such that $\nabla \cdot \langle\mathscr{U}\rangle_{(n+1)\Delta t} = 0$, as in Chorin-Temam projection method :

$$\begin{cases} \triangle p = \frac{1}{\Delta t}\nabla \cdot \langle\widetilde{\mathscr{U}}\rangle_{(n+1)\Delta t}, \; x \in \mathcal{D}, \\ \frac{\partial p}{\partial n_{\mathcal{D}}} = 0, \; \text{on } \partial\mathcal{D} \end{cases}$$

$\mathrm{U}^{j,N_p}_{(n+1)\Delta t} = \widetilde{\mathrm{U}}^{j,N_p}_{(n+1)\Delta t} - \Delta t \nabla p(\mathrm{X}^{j,N_p}_{(n+1)\Delta t})$ and $\langle\mathscr{U}\rangle_{(n+1)\Delta t} = \langle\widetilde{\mathscr{U}}\rangle_{(n+1)\Delta t} - \Delta t\nabla p$.

Convergence rate of the particle method for SLM

PhD Thesis of Laurent Violeau (Inria Tosca).



Cells of size ε for N particles in $\mathcal{D}$.

$$\langle F(\mathrm{U})\rangle(t,x) \simeq \frac{1}{N}\sum_{k=1}^{N} F\left(\mathrm{U}_t^{k,N}\right) K(\mathrm{X}_t^{k,N},x) \Big/ \frac{1}{N}\sum_{k=1}^{N} K(\mathrm{X}_t^{k,N},x)$$

$$\sum_{k=1}^{N} K(\mathrm{X}_t^{k,N},x) = N_{pc}$$

Lemma (in Violeau thesis)

For any $p > 1$ sufficiently small and $c > 0$, there exists C such that for all $\alpha > 0$, $\varepsilon > 0$ and $N > 1$ satisfying, $\dfrac{1}{\alpha^2 \varepsilon^d N^{\frac{1}{p}}} \leq c$, we have

$$\mathbb{E}\left[\left|B_\varepsilon[X_t^{j,N};\rho_t^\varepsilon] - B_\varepsilon[X_t^{j,N};\bar{\mu}_t^N]\right|\right] \leq C\left(\varepsilon + \frac{1}{\alpha\varepsilon^d N} + \frac{1}{\varepsilon^{d+1}N} + \frac{1}{(\varepsilon^d N)^{\frac{1}{p}}} + \frac{1}{\varepsilon^{\frac{dp}{2}}\sqrt{N}}\right).$$

If we choose $\alpha = \varepsilon$, the optimal rate of convergence is achieved for $N = \varepsilon^{-(d+2)p}$ and

$$\mathbb{E}\left[\left|B_\varepsilon[X_t^{j,N};\rho_t^\varepsilon] - B_\varepsilon[X_t^{j,N};\bar{\mu}_t^N]\right|\right] \leq C N^{-\frac{1}{(d+2)p}}.$$

Convergence rate of the particle method for SLM

$$\begin{cases} X_t = X_0 + \int_0^t U_s ds \text{ modulo } \mathbf{1}, \\ U_t = U_0 - \int_0^t \nabla V(X_s)\, ds + \int_0^t B[X_s, U_s; \rho_s]\, ds + W_t, \quad \text{for all } t \leq T, \\ \quad \rho_t \text{ is the density of } (X_t, U_t) \end{cases}$$

where B is the non linear conditional operator and V is a potential.
For the numerical test case :

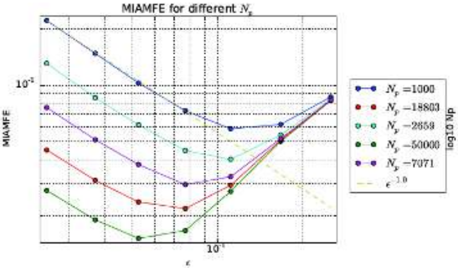
$$B[x, u; \gamma] = \frac{\int_{\mathbb{R}^d} (v - 2u)\gamma(x, v)\, dv}{\int_{\mathbb{R}^d} \gamma(x, v)\, dv},\ \forall (x, u) \in [0, 1]^2 \times \mathbb{R}^2,\ \gamma \in L^1([0, 1]^2 \times \mathbb{R}^2)$$

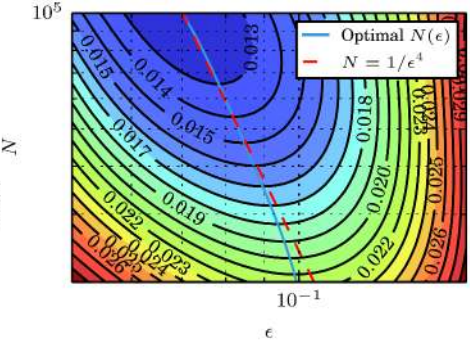
$$V(x_1, x_2) = \theta \cos(2\pi x_1) \sin(2\pi x_2) - \beta x_1,\ \forall x \in [0, 1]^2.$$

Note that in this case,

$$B[x, u; \rho_s] = \mathbb{E}[U_s | X_s = x] - 2u.$$

First rate of convergence result on the fluid particle algorithm for various cases of conditional expectation estimators.





The turbulent model in SDM

$$\begin{cases} dX_t = \mathrm{U}_t dt \\ d\mathbf{u}_t^{(i)} = -\partial_{x_i}\langle \mathscr{P}\rangle(t,X_t)dt + \left(\sum_j G_{ij}\left(\mathbf{u}^{(j)} - \langle \mathbf{u}^{(j)}\rangle\right)\right)(t,X_t)dt + \sqrt{C_0\varepsilon(t,X_t)}dB_t^{(i)} \\ \mathrm{U}_t = (\mathbf{u}_t^{(i)}, i = 1,2,3) \end{cases}$$

$$G_{ij} = -\frac{C_R}{2}\frac{\varepsilon}{k}\delta_{ij} + C_2\frac{\partial\langle \mathbf{u}^{(i)}\rangle}{\partial x_j} \qquad C_0 = \frac{2}{3}\left(C_R + C_2\frac{\mathcal{P}}{\varepsilon} - 1\right),$$

where $\mathcal{P}$ is the turbulent production term

$$\mathcal{P} = \frac{1}{2}\left(\mathcal{P}_{11} + \mathcal{P}_{22} + \mathcal{P}_{33}\right)$$

computed with

$$\mathcal{P}_{ij} := -\sum_k \left(\langle \mathbf{u}^{(i)'}\mathbf{u}^{(k)'}\rangle \frac{\partial\langle \mathbf{u}^{(i)}\rangle}{\partial x_k} + \langle \mathbf{u}^{(j)'}\mathbf{u}^{(k)'}\rangle \frac{\partial\langle \mathbf{u}^{(j)}\rangle}{\partial x_k} \right).$$

ε is closed with a mixing lenght or a turbulent viscosity parametrization.

Wall-boundary condition

The vertical component is simply reflected at z_{mirror} :

$$w_{\text{in}} = -w_{\text{out}}$$

$$u_{\text{in}} = u_{\text{out}} - 2\frac{\langle u'w'\rangle}{\langle w'^2\rangle}\, w_{\text{out}},$$

$$v_{\text{in}} = v_{\text{out}} - 2\frac{\langle v'w'\rangle}{\langle w'^2\rangle}\, w_{\text{out}}.$$

The covariances $\langle u'w'\rangle$ and $\langle v'w'\rangle$ are fixed to

$$\langle u'w'\rangle(t,x,y) = -\left(\frac{\langle u\rangle}{\sqrt{\langle u\rangle^2 + \langle v\rangle^2}} {u_*}^2\right)(t,x,y)$$

$$\langle v'w'\rangle(t,x,y) = -\left(\frac{\langle v\rangle}{\sqrt{\langle u\rangle^2 + \langle v\rangle^2}} {u_*}^2\right)(t,x,y)$$

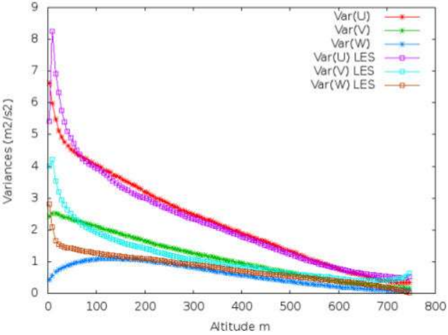
with $u_*(t,x_c,y_c) = \kappa \dfrac{\sqrt{\langle u\rangle^2(t,x_c,y_c,z_c) + \langle v\rangle^2(t,x_c,y_c,z_c)}}{\log\left(z_c/z_0\right)}$

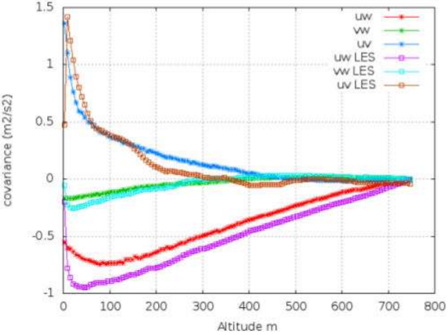
We also add elliptic blending model.

Validation

collaboration with P. Drobinski, IPSL

Comparison with a LES method [Drobinski etal 2007].
Renormalized variances and covariances

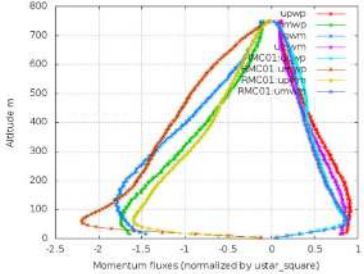


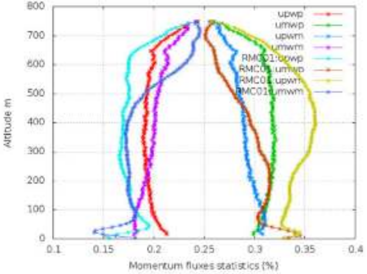


Validation

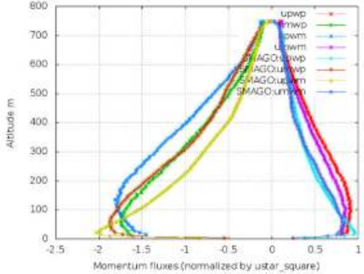
collaboration with P. Drobinski, IPSL

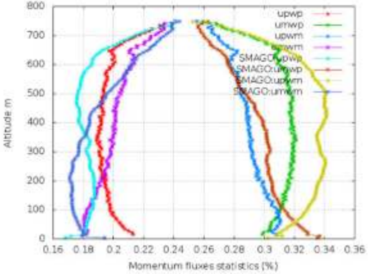
comparison with RMC01





comparison with SMAGO





Outline

- ▷ Wellposedness
- ▷ Algorithms and numerical analysis
- ► Application : computation of wind circulation around mills

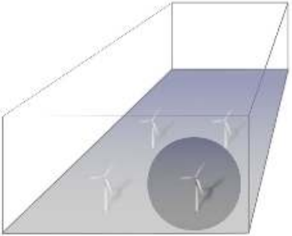
Actuator disc methods in the Lagrangian setting

$$
\begin{aligned}
d\mathrm{X}_t =& \mathrm{U}_t dt \\
d\mathrm{U}_t =& \left(-\frac{1}{\rho} \nabla_x \langle \mathscr{P} \rangle (t, \mathrm{X}_t) \right) dt \\
& - G(t, \mathrm{X}_t) \Big(\mathrm{U}_t - \langle \mathrm{U} \rangle (t, \mathrm{X}_t) \Big) dt + C(t, \mathrm{X}_t) dW_t \\
& + f\,(t, \mathrm{X}_t, \mathrm{U}_t)\, dt + f_{\mathrm{nacelle}}\,(t, \mathrm{X}_t, \mathrm{U}_t)\, dt + f_{\mathrm{mast}}\,(t, \mathrm{X}_t, \mathrm{U}_t)\, dt.
\end{aligned}
$$

$f(t, \mathrm{X}_t, \mathrm{U}_t)$ the body forces that the blades exert on the flow.

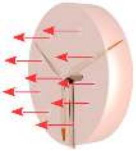
Supplementary terms $f_{\mathrm{nacelle}}(t, \mathrm{X}_t, \mathrm{U}_t)$ and $f_{\mathrm{mast}}(t, \mathrm{X}_t, \mathrm{U}_t)$ represent the impact of the mill nacelle and mast.

Turbine wake effects

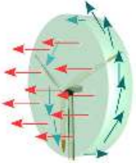


- Forces are computed using physical parameters of the turbines (Actuator Disc Theory/Blade Element Method).
- Wake propagation **is computed as part of the main simulation.**
- **Incorporates turbulence effects in the wake.**
- Different force models are available:

Uniformly loaded Actuator Disc

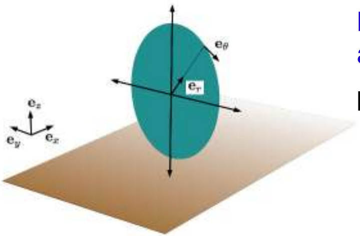


Rotating Actuator Disc



Actuator Line

Forces



Non rotating, uniformly loaded actuator disc model :

knowing the *axial induction factor* a

$$f_x = -\frac{1}{\Delta x}\frac{2a}{1-a}\left|\mathbb{E}\left[u_t / \mathrm{X}_t \in \mathcal{C}\right]\right|^2 \mathbf{e}_x.$$

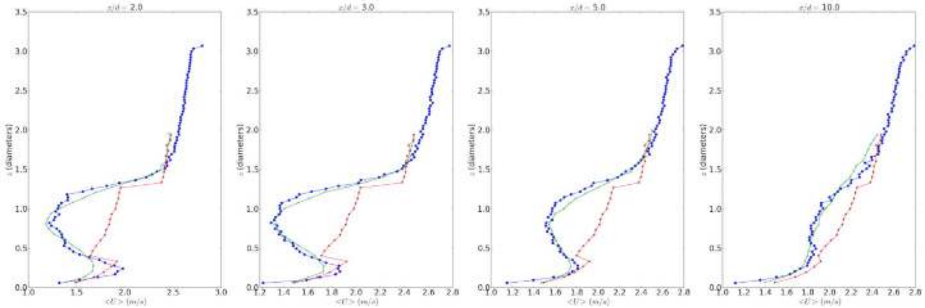
Rotating actuator disc :

$$f_x(t, \mathrm{X}_t, \mathrm{U}_t) = -\mathbb{1}_{\{\mathrm{X}_t \in \mathcal{C}\}} \frac{N_b}{4\pi r \Delta x}\left(U_{\mathrm{relat}}(\mathrm{X}_t, \mathrm{U}_t)\right)^2 c(r(\mathrm{X}_t))\left(C_L(\alpha)\cos(\phi) + C_D(\alpha)\sin(\phi\right.$$

$$f_\theta(t, \mathrm{X}_t, \mathrm{U}_t) = \mathbb{1}_{\{\mathrm{X}_t \in \mathcal{C}\}} \frac{N_b}{4\pi r \Delta x}\left(U_{\mathrm{relat}}(\mathrm{X}_t, \mathrm{U}_t)\right)^2 c(r(\mathrm{X}_t))\left(C_L(\alpha)\sin(\phi) - C_D(\alpha)\cos(\phi)\right)$$

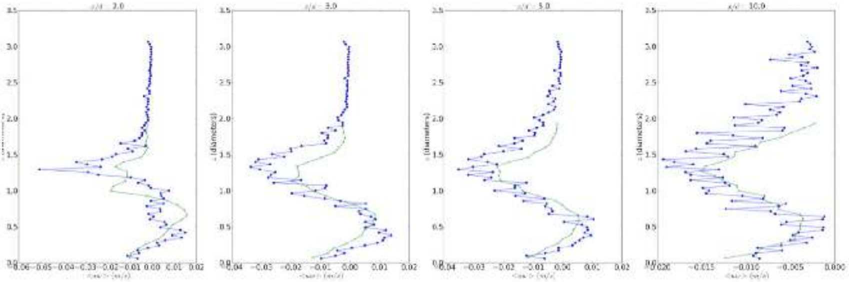
Validation with wind tunnel measures

With actuator line model.



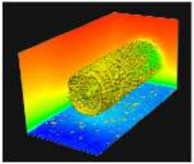
blue : is SDM simulation
green : wind tunnel measurements [from Wu-Porte-Agel 2011]
red : comparison with Jensen model.

Validation with wind tunnel measures

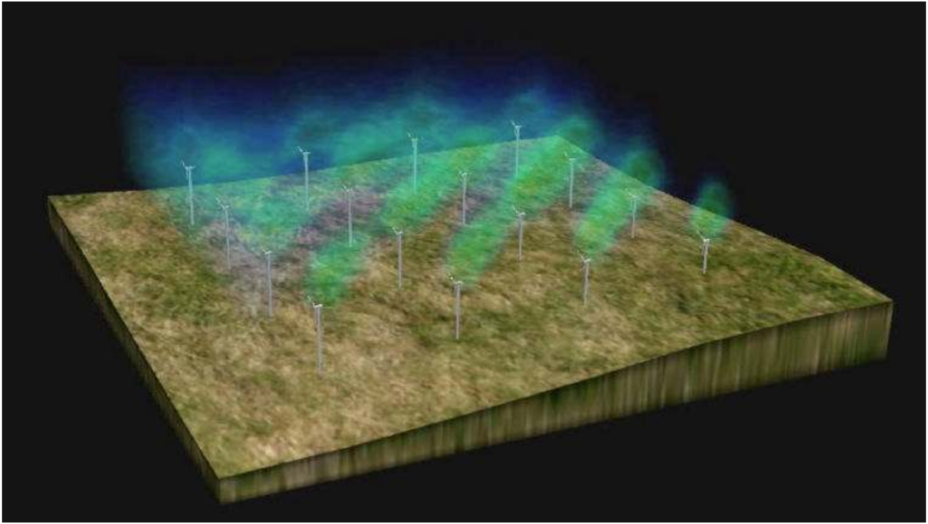


blue is SDM simulation
green is wind tunnel measurements [from Wu-Porte-Agel 2011]

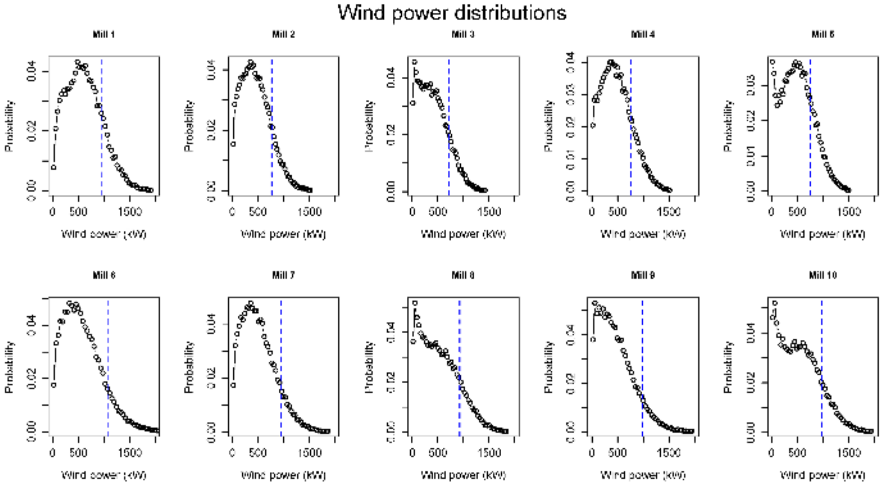
Vorticity structure



Wind farm evaluation



Wind farm evaluation



Collaborative works with

Jean Francois Jabir (University of Valpareiso)
Joaquin Fontbona (U Chile)
Pierre Emmanuel Jabin (U Maryland)

Antoine Rousseau (Inria Lemon)
Philippe Drobinski (LMD-IPSL)

Laurent Violeau (Phd student Inria)
José Espina, Jacques Morice, Cristian Paris (Inria Chile)
Selim Kraria (Inria SED)

Thank you

www-sop.inria.fr/members/Mireille.Bossy

Bibliography

F. Bernardin, M. Bossy, C. Chauvin, P. Drobinski, A. Rousseau, and T. Salameh.
Stochastic Downscaling Methods : Application to Wind Refinement.
Stoch. Environ. Res. Risk. Assess., 23(6), 2009.

F. Bernardin, M. Bossy, C. Chauvin, J-F. Jabir, and A. Rousseau.
Stochastic Lagrangian Method for Downscaling Problems in Computational Fluid Dynamics.
ESAIM : M2AN, 44(5) :885–920, 2010.

M. Bossy, J. Espina, J. Morice, C. Paris, and A. Rousseau.
Modeling the wind circulation around mills with a lagrangian stochastic approach.
Preprint arXiv :1404.4282, 2014.

M. Bossy, J. Fontbona, P-E. Jabin, and J-F. Jabir.
Local existence of analytical solutions to an incompressible lagrangian stochastic model in a periodic domain.
Communications in Partial Differential Equations, 38(7) :1141–1182, 2013.

M. Bossy and J-F. Jabir.
On confined mckean langevin processes satisfying the mean no-permeability boundary condition.
Stochastic Processes and their Applications, 121(12) :2751 – 2775, 2011.

M. Bossy, J.F. Jabir, and D. Talay.
On conditional McKean Lagrangian stochastic models.
Probab. Theory Relat. Fields, 151 :319–351, 2011.

Mireille Bossy and Jean-François Jabir.
Lagrangian stochastic models with specular boundary condition.
Preprint arXiv :1304.6050, 2013.

L. Violeau.
Particles method for Lagrangian Stochastic Models.
PhD thesis, PhD Thesis, Université de Nice Sophia Antipolis