

Calibrating a hydrological model robustly to rain perturbations with stochastic surrogates

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Context

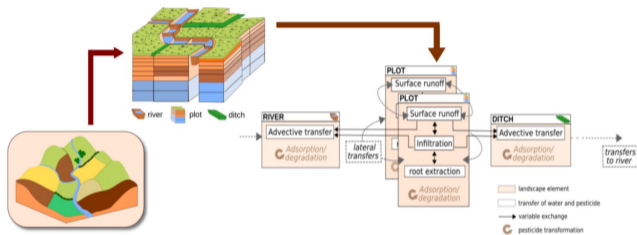
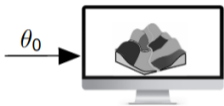


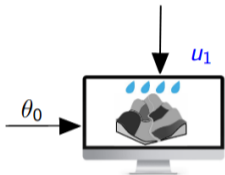
Figure: The PESHMELBA model, Rouzies, E., Lauvernet, C., Barchet, C., Morel, T., Branger, F., Braud, I., Carlier, N. (2019). From agricultural catchment to management scenarios: A modular tool to assess effects of landscape features on water and pesticide behavior. *Science of The Total Environment*, 671, 1144–1160. <https://doi.org/10.1016/j.scitotenv.2019.03.060>.

- water and pesticide transfer model
- spatially distributed
- many parameters to be precised
- a part of them cannot be measured directly
- => calibrate the parameters with terrain observations

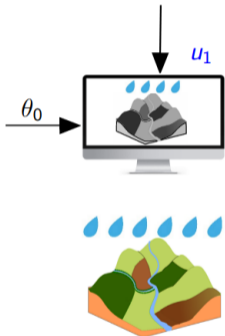
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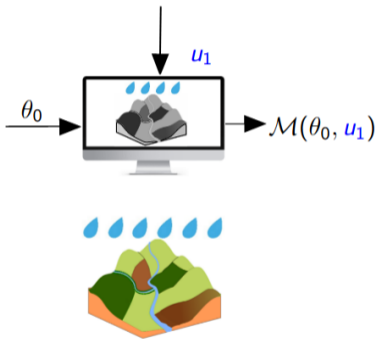
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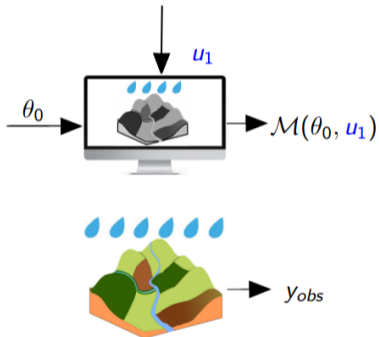
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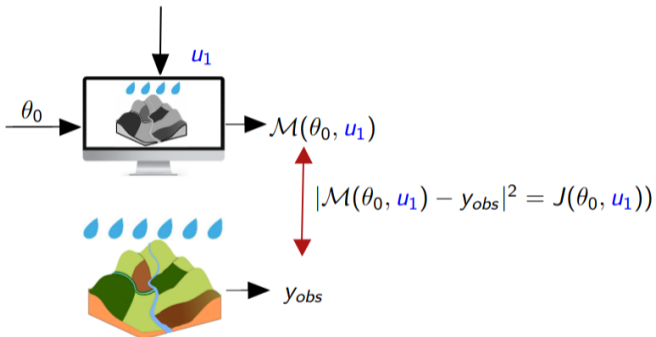
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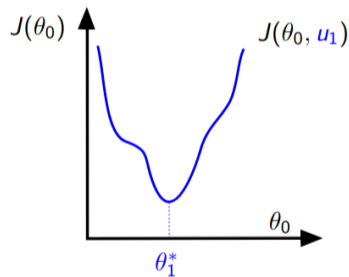
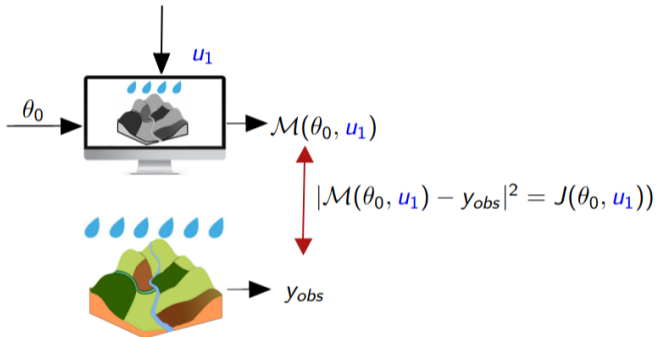
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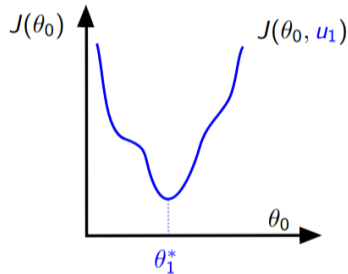
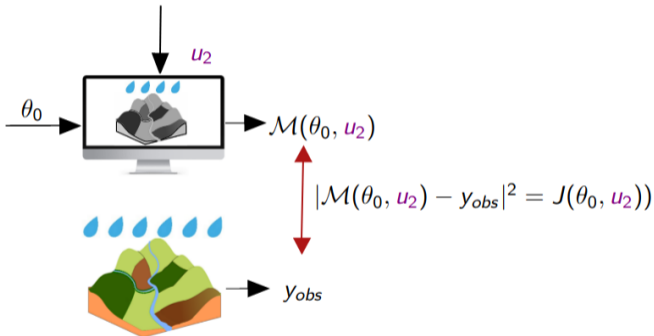
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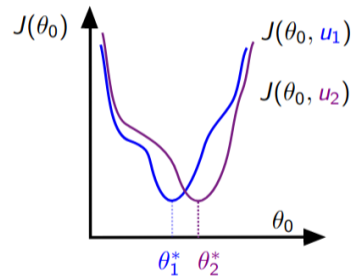
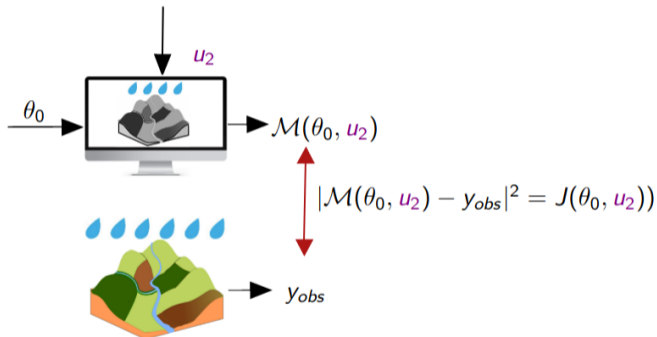
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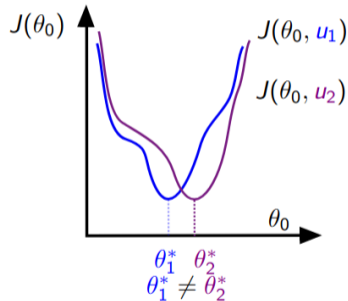
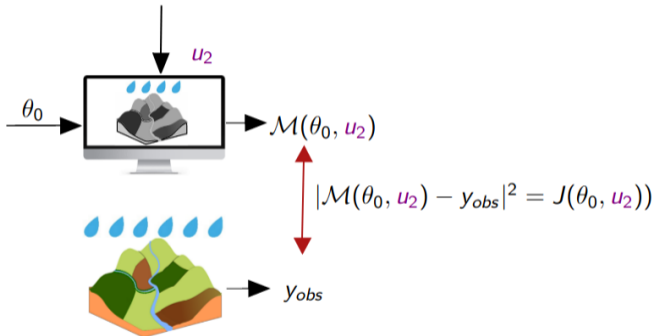
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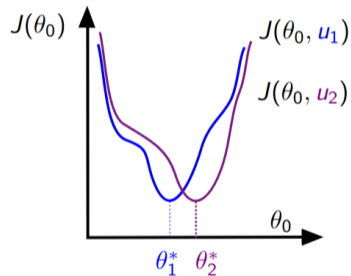
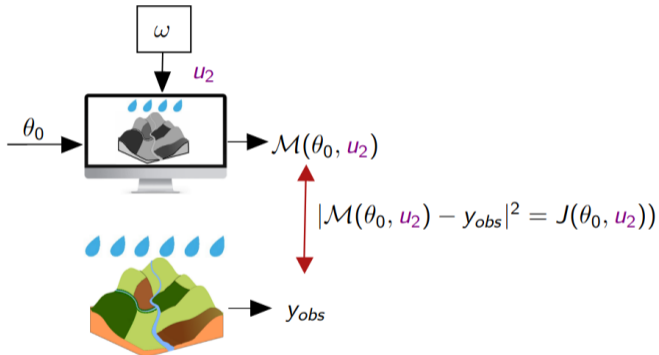
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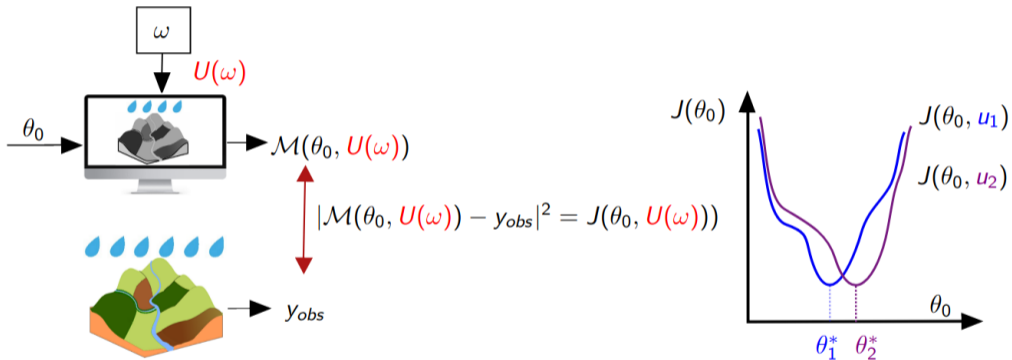
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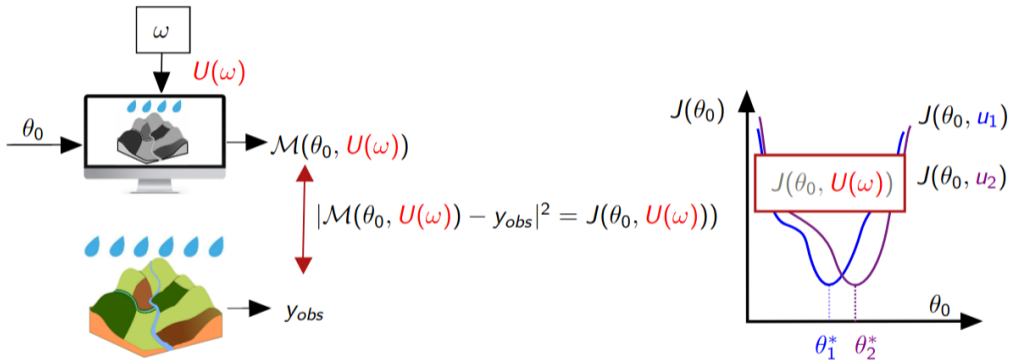
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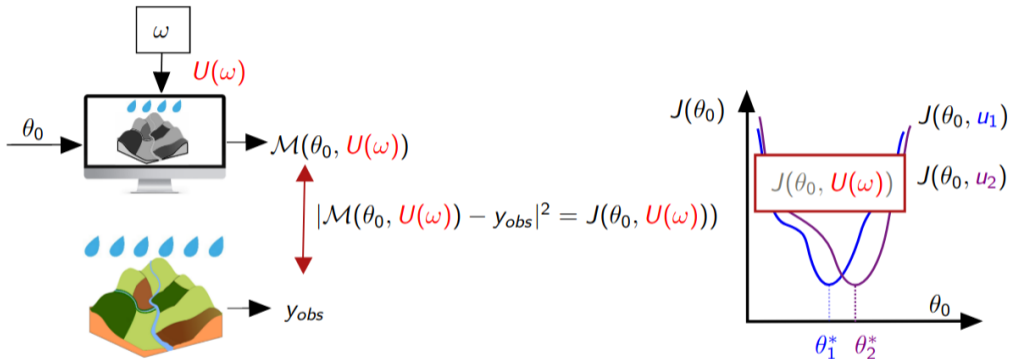
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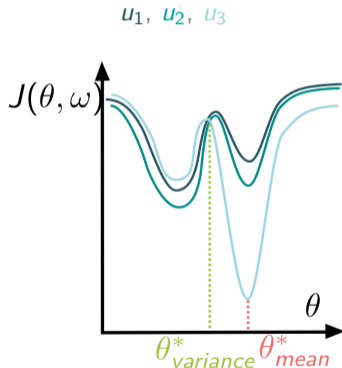


Introduction



What does it mean to find a *robust* minimizer ?

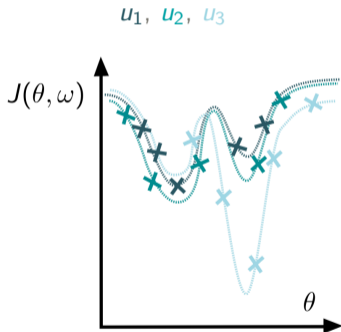
Methods: Robust estimators



1. $\theta_{\mathbb{E}}^* = \underset{\theta}{\operatorname{argmin}} \mathbb{E}_U[J(\theta, U)],$
2. $\theta_{\text{Var}}^* = \underset{\theta}{\operatorname{argmin}} \text{Var}_U[J(\theta, U)],$
3. Pareto of the two
4. ...

- What is the probability space of $U(\omega)$?

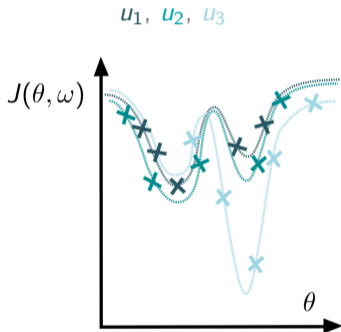
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- How to estimate the robust parameters from a limited number of model simulations?

Methods: Robust estimators



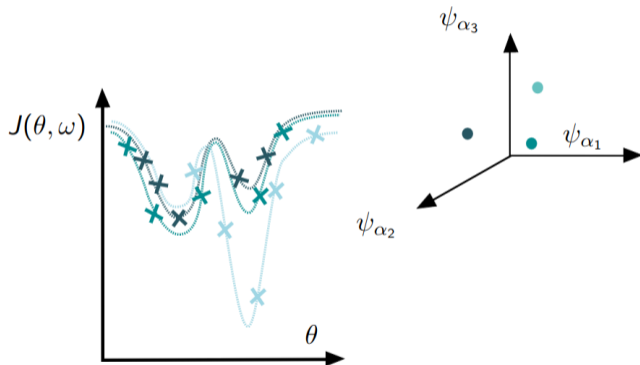
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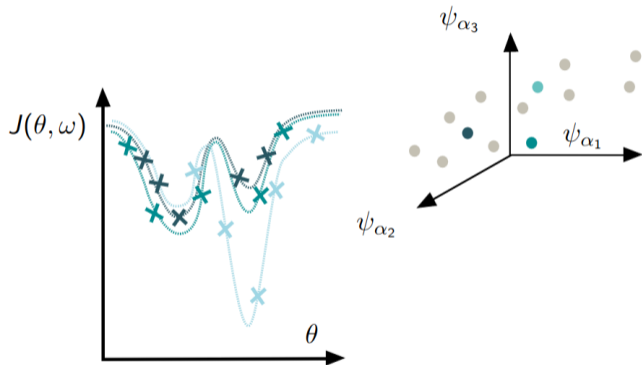
→ Stochastic metamodel

N. Lüthen, S. Marelli, B. Sudret: A spectral surrogate model for stochastic simulators computed from trajectory samples, 2023. DOI : 10.1016/j.cma.2022.115875

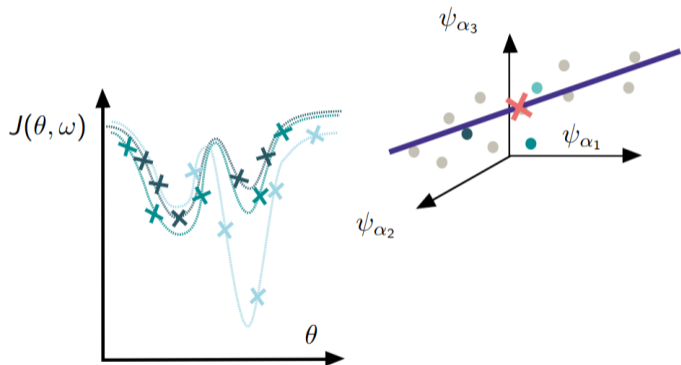
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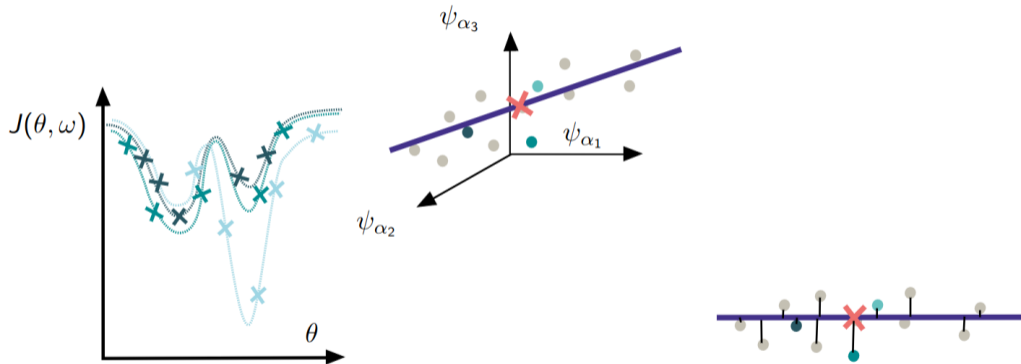
Methods: Stochastic metamodel



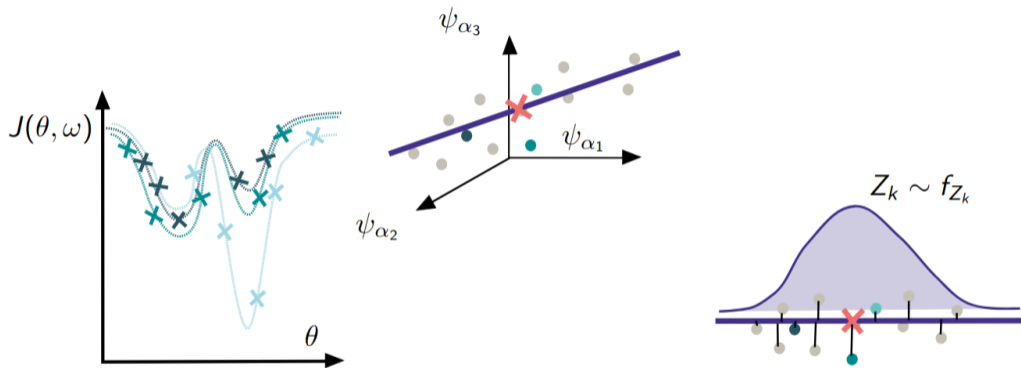
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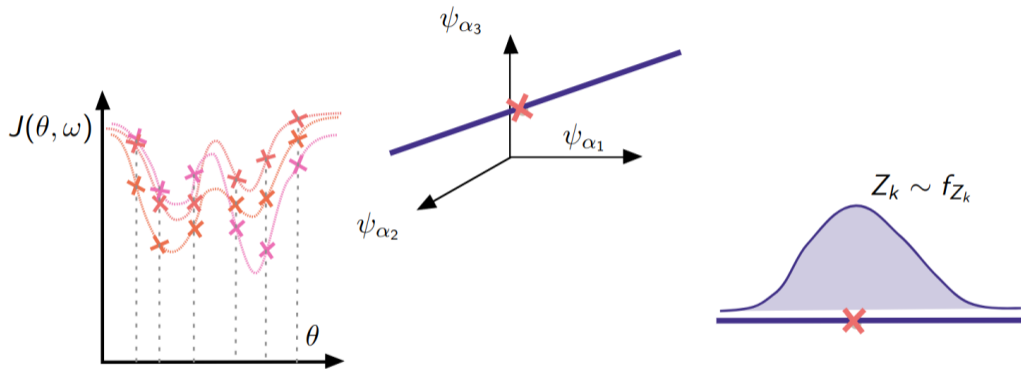


Methods: Stochastic metamodel



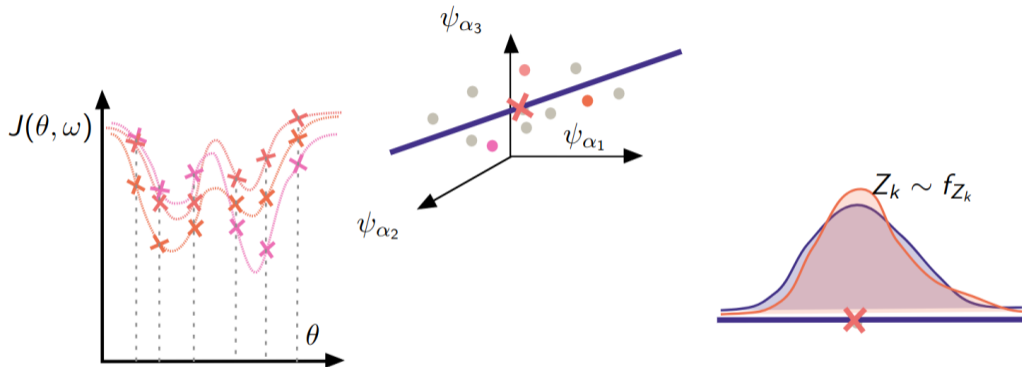
$$\hat{J}(\theta, U(\omega)) = \hat{\mu}(\theta) + \sum_{k=1}^K \sqrt{\lambda_k} Z_k(\omega) \left(\sum_{\alpha \in \mathcal{A}} b_{\alpha}^{(k)} \psi_{\alpha}(\theta) \right)$$

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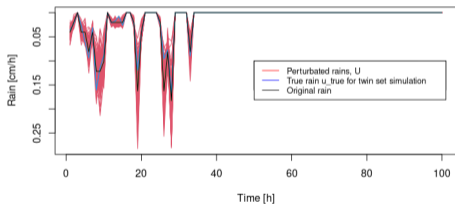
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Case study: Moisture profile observations

$$\theta \in \mathbb{R}^2, y_{true} \in \mathbb{R}^{25}$$

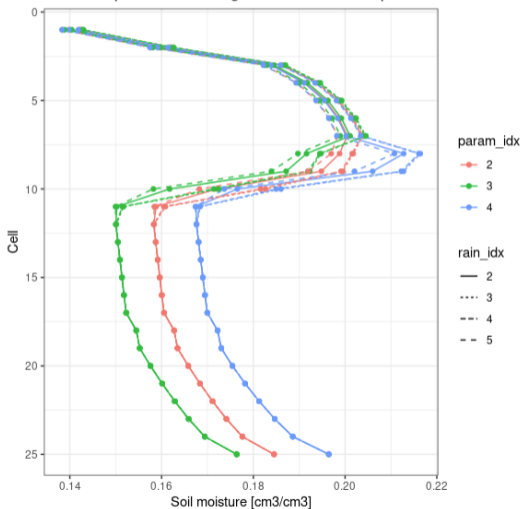
$$J(\theta, u) = (\mathcal{M}(\theta, u) - y_{true})^2$$

Perturbed rains



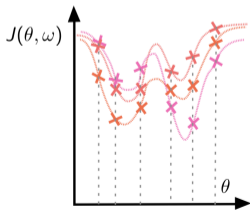
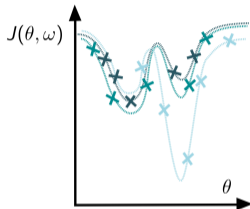
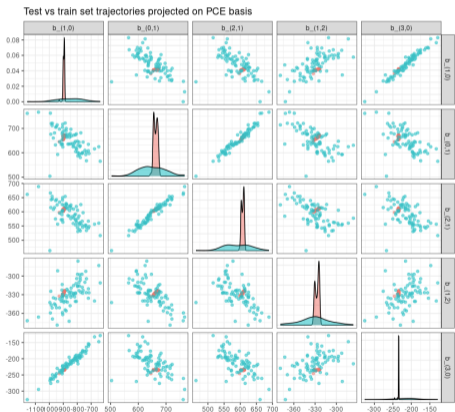
The impact of rain perturbations on the observations is very small.

Rain and parameter changes on the moisture profile



Results: Validation of the emulator

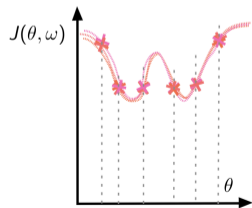
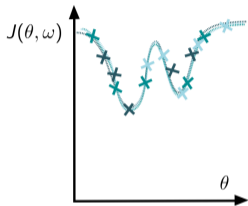
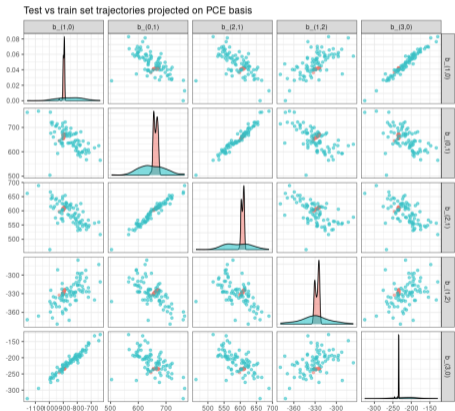
Compare **train** and **test** trajectories of PESHMELBA simulations, projected on the PCE basis.



The **train** set and the **test** set do not present the same variabilities, thus the emulator does not reproduce correctly the impact of rain perturbations on the cost function.

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Improvements on the emulator ?

- augment rain perturbations on the cost function (true rain measurements on the Yzeron catchment)
- use a training set, where all the experimental designs on each rain are the same
- observe another output, more sensible to rain perturbations than moisture profiles

Thank you for your attention !