An adapted derivative-free optimization method for the optimal design of turbine blades

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Abstract:

The motivating application of this work is the optimal design of the turbine blades of an helicopter engine [1]. The objective is to minimize the vibrations of the compressor which are obtained as outputs of a very expensive mechanical simulations (several hours for one design) by selecting the arrangement of two pre-defined blade shapes on the turbine disk (Figure 1, left). In this context, some continuous parameters describe the blade shapes (e.g. thickness, length of the blades ...) and additional binary variables locate the two pre-defined blade geometries on the disk. Note that the cyclic property of the problem yields to a huge number of redundant arrangements: two bladed disks that differ only by a rotation of the pattern lead to the same value of the objective function.
The optimization problem is stated as the following mixed binary black-box problem

$$\begin{cases} \min\limits_{x,y} f(x,y) \\ x \in [\underline{x}, \bar{x}], y \in \{0,1\}^n, \end{cases}$$

where $x \in \mathbb{R}^m, y \in \{0,1\}^n$ are respectively the continuous and binary blade design variables and f is the objective function given as the output of the "black-box" simulator modelling the vibration amplitude.

In this study, we focus on the class of derivative free trust region methods [2, 3] and their extension to mixed binary problems proposed by [4]. This method, denoted DFOb-TR in the sequel, is based on successive quadratic optimization problems which rely on local interpolation models valid within an adaptive trust region defined as

$$B(x_c, y_c, \Delta_x, \Delta_y) = \{x \in \mathbb{R}^m, y \in \{0,1\}^n \mid \|x - x_c\|_\infty \le \Delta_x \text{ and } \|y - y_c\|_H \le \Delta_y\},$$

with (x_c, y_c) and (Δ_x, Δ_y) the center and the size of the region respectively. $\|.\|_\infty$ is the infinite norm and $\|.\|_H$ the Hamming distance. To force exploration for binary variables, a "no-good-cut" constraint [5] based on the Hamming distance is introduced

$$\sum_{j:y_{c_j}=0} y_j + \sum_{j:y_{c_j}=1} (1 - y_j) \ge k_c,$$

where $y_{c_j}, j = 1, \ldots, n$ are the binary components of the center and $k_c \in \mathbb{N}^*$ is the size of the explored region to be excluded in exploration phase.

As already emphasized, to be efficient, the optimization procedure should take into account the inherent cyclic symmetry of the problem. To this end, we introduce the necklace concept (Figure 1, right) [6] and model the bladed disk with two pre-defined blade geometries as a two-color necklace.

Thus, we propose an appropriate distance which detects similar (rotated) solutions, i.e. necklaces. This new distance replaces the Hamming distance for binary variables in the trust region definition and in no-good-cut constraints. Equivalent quadratic sub-problems are defined in order to be solved efficiently.

Numerical results obtained for benchmark functions and for a simplified version of Safran's application are presented. DFOb-TR method with necklace distance is compared with state-of-the-art black-box optimization methods NOMAD [7] and RBFopt [8].

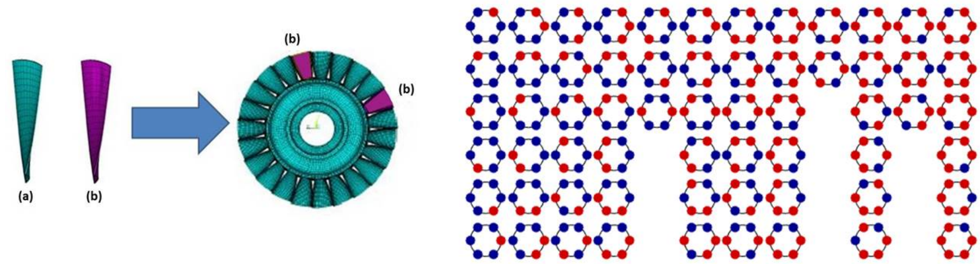

Figure 1: (Left) A blade configuration with two different pre-defined shapes (a) and (b) [1]. (Right) Each column represents similar necklaces obtained by rotation.

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Short biography – Thi-Thoi Tran got her bachelor's degree in mathematics from Hanoi university of sciences and then graduated with a Master's degree in optimization at Limoges University. She is currently a second year PhD student at IFPEN. The thesis, funded by Safran Tech, focuses on nonlinear optimization problem with mixed continuous and discrete variables for black-box simulators. A first industrial application is the optimal design of the turbine blades of an helicopter engine.